Electromagnetic Self-force and Overcharging a Reissner-Nordström Black Hole

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Cosmic Censorship

- The Weak Cosmic Censorship Conjecture posits that spacetime singularities are hidden behind event horizons, making them invisible to all observers in the external universe.

- Counterexamples:
  1. Critical collapse of a massless scalar field with infinitely fine-tuned initial conditions [Choptuik 1993].
  2. Gregory-Laflamme instability of black strings [Gregory and Laflamme 1993, Lehner and Pretorius 2010].
  3. Super-saturation of a black hole's parameters by overcharging/overspinning [Hubeny 1998, Jacobson and Sotiriou 2009].

Infalling test-charges in Reissner-Nördstrom

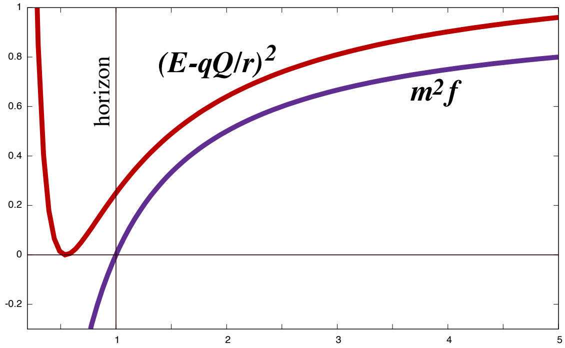
- The spacetime of a spherically symmetric charged black hole is described by the Reissner-Nördstrom (RN) line element

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \quad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$

- We consider a particle of mass m and charge $q \gg m$ following a radial path towards a RN black hole having mass M and charge $Q = M(1 - 2\epsilon^2)$ for small positive ϵ.

- The RN spacetime admits a timelike Killing vector $t^\alpha = \frac{\partial x^\alpha}{\partial t}$ that gives rise to a conserved energy $E_0 = -t^\alpha(mu_\alpha + qA_\alpha)$.

Infalling Test-charges in Reissner-Nordström



- Overcharging occurs when $Q + q > M + E_0$.
- A test charge with an open set of allowed parameters $\{q, m, E_0\}$ moving according to $m^2 \dot{r}^2 = (E_0 - qQ/r)^2 - m^2 f(r)$ can cross the event horizon and overcharge a near-extreme black hole.

Overcharging Conditions

- The overcharging conditions are
  1. $\dot{r}^2 > 0\,, \quad \forall r \geq r_+$
  2. $Q + q > M + E_0$
- By setting $M \equiv 1$, $Q \equiv 1 - 2\epsilon^2$, Hubeny showed that the parameter space for overcharging is the three-parameter family characterized by

$$
\begin{aligned}
q &= a\epsilon & a &> 1\,,\\
E_0 &= a\epsilon - 2b\epsilon^2 & 1 < b &< a\,,\\
m &= c\epsilon & c &< \sqrt{a^2 - b^2}\,.
\end{aligned}
$$

Including Self-Force Effects

- Hubeny's overcharging condition fails to account for energy lost by the particle due to radiation, which is also $O(\epsilon^2)$.
- The full $O(\epsilon^2)$ overcharging condition reads

$$q + Q > M + E_0 - E_{\text{rad}},$$

where E_{rad} is the energy radiated to null future infinity.

- In addition, the radial acceleration of a Hubeny orbit at the event horizon is $O(\epsilon^2)$. Therefore self-force corrections, which are also of order ϵ^2, must be included in the infall condition.
- The self-force is incorporated by computing the work it does on the particle as it moves inward

$$E(r) = E_0 - q \int_r^\infty F_{tr}^{\text{self}} \, \mathrm{d}r$$

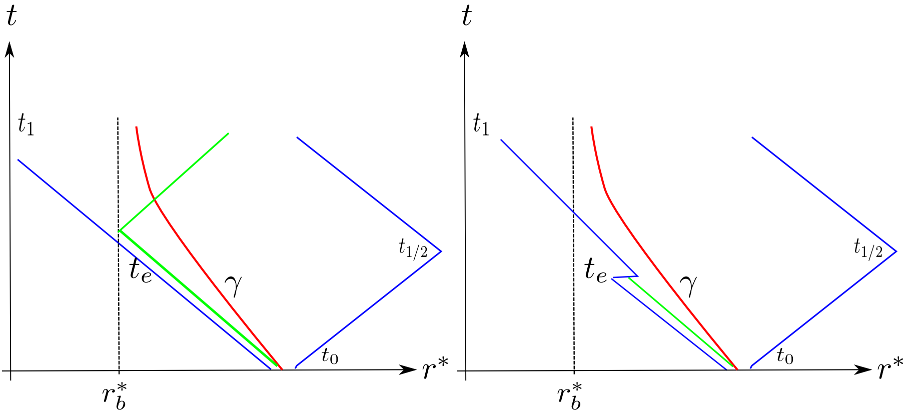
- The infall condition in the presence of the self-force is

$$(E(r) - qQ/r)^2 > m^2 f(r)\,, \quad \forall r \geq r_+$$

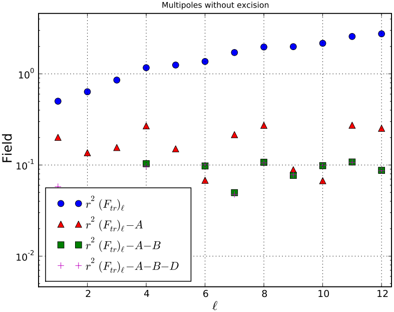
The Initial Data Struggle

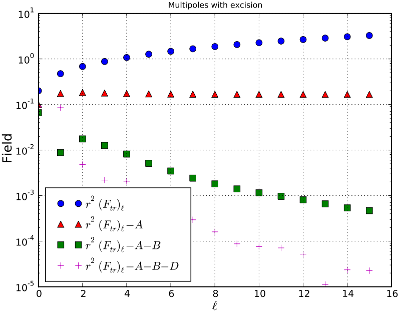
- We evolve the multipoles of the retarded field using a $1+1$ time-domain scheme with trivial initial data.
- The use of unphysical/inconsistent initial data creates a burst of "junk radiation".
- Typical overcharging orbits are very high speed. In these cases, where the particle and the ingoing junk radiation are traveling in close proximity, the self-force at the particle is severely contaminated.
- We've tried several things to minimize the effect of the junk: static initial data, adiabatic transition of the source, transition of the trajectory, and direct excision of the ingoing null ray. Direct excision proved to be the most effective.

Grid Excision

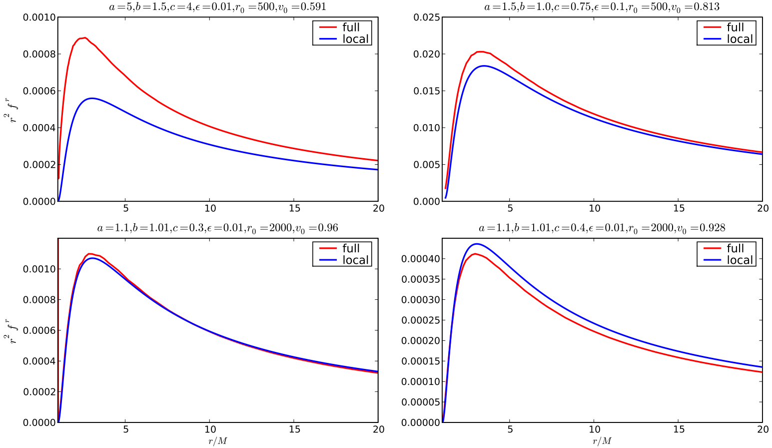


Excision Results





Self-Force Results



- Overall, the local self-force $f^\alpha_{\rm loc} = \frac{2}{3} q^2 \frac{{\rm D}a^\alpha}{{\rm d}\tau}$ captures the general features of the full self-force.

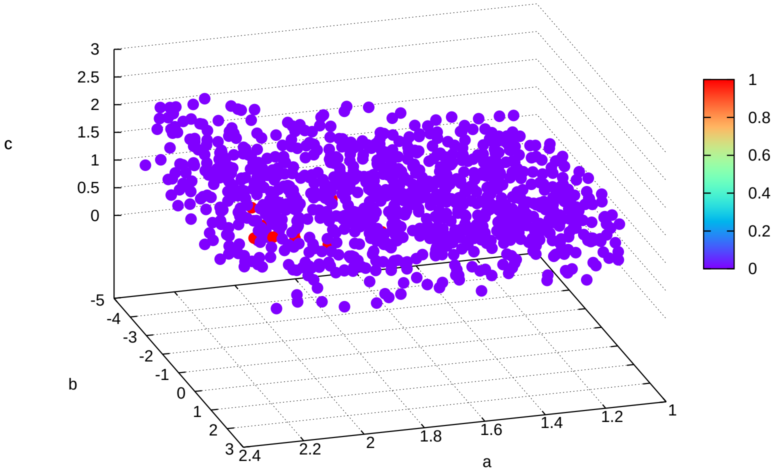
Searching for Overcharging Candidates

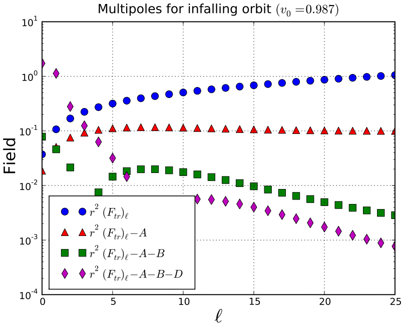
- To seach for overcharging candidates we randomly generate parameters in the $\{a, b, c\}$ space and implement a two-step selection procedure:
  1. We use the local self-force to select only the orbits for which the particle crosses the event horizon.
  2. From these orbits we select only those whose parameters satisfy the overcharging condition

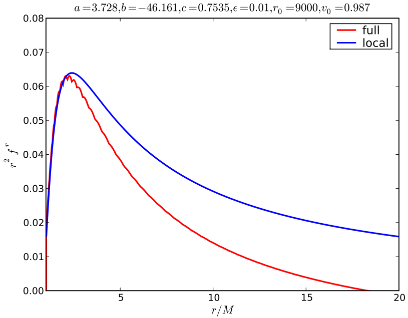
     $$q + Q > E_0 - E_{\mathsf{rad}} + M$$

     where E_{rad} is approximated using the Larmor formula

     $$\frac{\mathrm{d}E_{\mathsf{rad}}}{\mathrm{d}t} \approx \frac{2}{3}\frac{q^2}{m^2} F^{\mu}_{\mathsf{BH}} F_{\mathsf{BH}\,\mu}$$


c
3
2.5
2
1.5
1
0.5
0
-5
-4
-3
-2
-1
0
1
2
3
b
2.4
2.2
2
1.8
1.6
1.4
1.2
1
a
1
0.8
0.6
0.4
0.2
0





For this candidate the full self-force is not sufficient to turn the particle around. The flux is required to determine if the particle actually overcharges the BH.

Conclusions and Future Work

- A Monte Carlo scan of the parameter space, based on crude estimates of the flux and the self-force, reveals candidate trajectories that may overcharge the BH.
- These cases (and others) require fuller scrutiny with exact calculations of the flux and self-force.
- Thanks to the excision trick, techniques to carry out the computations are in hand.
- Flux calculations are underway and answers will be forthcoming in the next weeks.
- An argument, based on extended bodies and the third law of BH mechanics is being devised which suggests that the self-force must enforce cosmic censorship.

Appendix I: Particle Motion

- The equation of motion for a particle of mass m and charge q moving under the influence of an external electromagnetic field $F_{\alpha\beta}^{\text{ext}}$ is given by

$$ma_\alpha = qF_{\alpha\beta}^{\text{ext}}u^\beta.$$

- Radial, accelerated motion in Reissner-Nordström spacetime is described by the equation

$$\frac{\mathrm{d}^2 r^*}{\mathrm{d}t^2} = -\frac{m^2 f}{(E_0 r - qQ)^3}\Big(rME_0 + qQ(M-r) - Q^2 E_0\Big),$$

where

$$E_0 = \sqrt{\dot{r}^2 + f} + qQ/r$$

in Schwarzschild coordinates $x^\alpha = \{t, r, \theta, \phi\}$.

Appendix II: Retarded Field

- The retarded field $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ obeys the sourced Maxwell Equation

$$g^{\mu\beta}\nabla_\beta F_{\alpha\mu} = 4\pi j_\alpha,$$

where A_α is the electromagnetic potential and

$$j_\alpha = q\int u_\alpha\,\delta_4(x, z(\tau))d\tau.$$

- Spacetime tensors are written as multipole expansions in spherical harmonics which decouple the radial and temporal tensor components from angular ones.
- Radial motion is imposed which sets both the m-modes and the angular current components to zero.

Appendix III: Retarded Field

- For radial motion, the relevant field multipole is

$$F^{\ell}_{tr} = -\frac{1}{r^2}\psi^{\ell}(t,r)Y^{\ell}(\theta^A),$$

and the non-zero source components are given by

$$j^{\ell}_t = -q\sqrt{\frac{2\ell+1}{4\pi}}\frac{f_0}{r_0^2}\delta(r - r_0(t)),$$
$$j^{\ell}_r = q\sqrt{\frac{2\ell+1}{4\pi}}\frac{dr_0/dt}{f_0 r_0^2}\delta(r - r_0(t)).$$

- The field ψ^{ℓ} obeys the wave equation

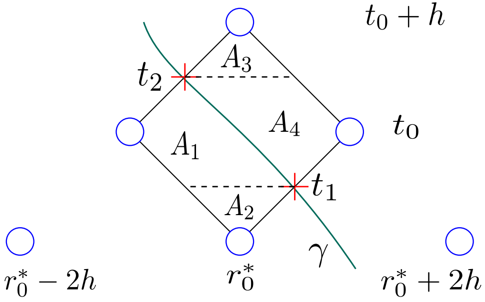
$$4\pi f\left[\partial_t(r^2 j^{\ell}_r) - \partial_r(r^2 j^{\ell}_t)\right] = -\partial^2_t\psi^{\ell} + f\partial_r(f\partial_r\psi^{\ell}) - \frac{\ell(\ell+1)}{r^2}f\psi^{\ell}.$$

Appendix IV: Retarded Field - Numerical Method

- The wave equation for ψ^ℓ is written in standard form

$$\left[\partial^2_{r^*} - \partial^2_t - V^\ell(r)\right]\psi^\ell(t,r) = S^\ell(r_0(t),r).$$

- The source cell is divided into sub-areas $A_{i=1\ldots4}$ based on the locations where the particle enters and leaves the cell: (t_1, r^*_1) and (t_2, r^*_2).



Appendix V: Retarded Field - Numerical Method

- The field is evolved according to the second-order algorithm of Lousto and Price

$$\begin{aligned}\psi_{\mathrm{vac}}(t+h,r^*) &= -\psi(t-h,r^*) + [\psi(t,r^*+h) \\ &\quad + \psi(t,r^*-h)]\left[1-\frac{1}{2}h^2V(r^*)\right] + O(h^4), \\ \psi_{\mathrm{source}}(t+h,r^*) &= -\psi(t-h,r^*)\left[1+\frac{V(r^*)}{4}(A_2-A_3)\right] \\ &\quad + \psi(t,r^*+h)\left[1-\frac{V(r^*)}{4}(A_3+A_4)\right] \\ &\quad + \psi(t,r^*-h)\left[1-\frac{V(r^*)}{4}(A_1+A_3)\right] \\ &\quad - \frac{1}{4}\left[1-\frac{V(r^*)}{4}A_4\right]\iint_{\mathrm{cell}} S\,\mathrm{d}u\,\mathrm{d}v + O(h^3).\end{aligned}$$

Appendix VI: Singular Field

- The potential A_α contains a piece which is singular on the world line.
- The multipole coefficients of the singular field constitute the regularization parameters A, B, C, and D.
- The regularization parameters are computed perturbatively using a local expansion about the world line. Their values are

$$
\begin{aligned}
A &= -\frac{1}{r_0^2}\mathrm{sign}(\Delta), \\
B &= -\frac{E}{2mr_0^2} + \frac{qQ}{mr_0^3}, \\
C &= 0, \\
D &= -\frac{3}{16}\frac{E(E-m)(E+m)}{m^3 r_0^2} + \frac{3}{4mr_0^3}\left(\frac{qQ(m^2+E^2)}{2m^2} - ME\right).
\end{aligned}
$$