Reduced-Basis Output Bounds:
Reliable Real-Time Solution of
Parametrized Partial Differential Equations

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We present a technique for the rapid and reliable prediction of linear-functional outputs of elliptic (and parabolic) partial differential equations with affine parameter dependence. The essential components are (*i*) (provably) rapidly convergent global reduced-basis approximations — Galerkin projection onto a space W_N spanned by solutions of the governing partial differential equation at N selected points in parameter space; (*ii*) *a posteriori* error estimation — relaxations of the error-residual equation that provide inexpensive yet sharp and rigorous bounds for the error in the outputs of interest; and (*iii*) off-line/on-line computational procedures — methods which decouple the generation and projection stages of the approximation process. The operation count for the on-line stage — in which, given a new parameter value, we calculate the output of interest and associated error bound — depends only on N (typically very small) and the parametric complexity of the problem; the method is thus ideally suited for the repeated and rapid evaluations required in the context of parameter estimation, design, optimization, and real-time control.

In this talk we first present the computational formulation and associated numerical analysis for coercive, linear problems; we then discuss newer results for noncoercive, nonlinear problems — including the steady incompressible Navier-Stokes equations.