Evolving Complex Networks:

The Backbone of the Climate Network

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Outline

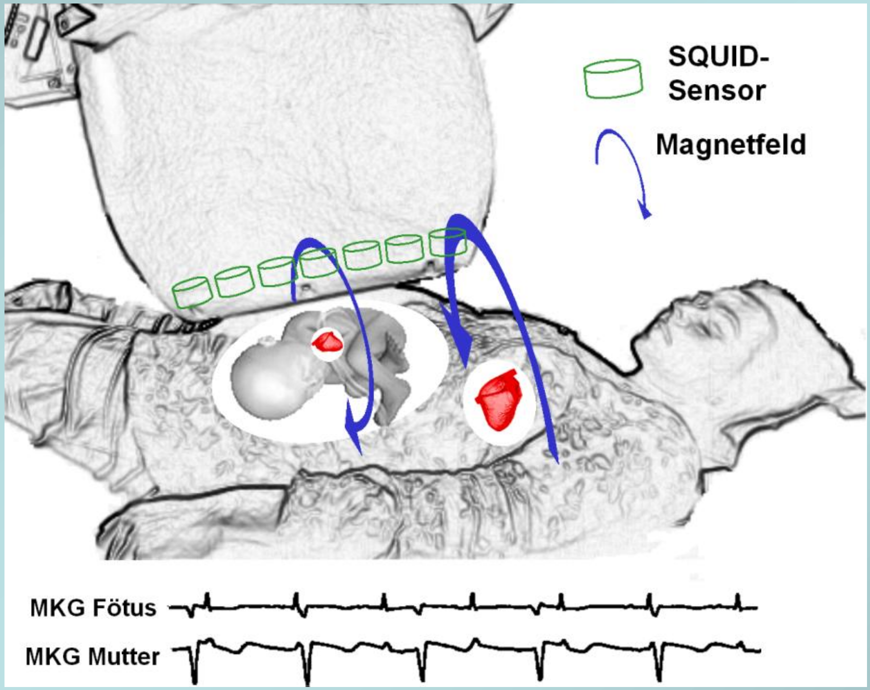
- Introduction and Synchronization
- Reconstruction of complex networks from climate dynamics
- Recurrence networks and their application to climate data
- Conclusions

Nonlinear Sciences and Synchronization

Start in 1665 by Christiaan Huygens:

Discovery of **phase synchronization,** called **sympathy**

Application: Mother-Fetus System



Magnetocardiography

Testing the foetal–maternal heart rate synchronization

Riedl M, van Leeuwen P, Suhrbier A, Malberg H, Grönemeyer D, Kurths J, Wessel N. Testing the fetal maternal heart rate synchronisation via model based analysis. **Philos Transact A Math Phys Eng Sci. 367, 1407 (2009)**

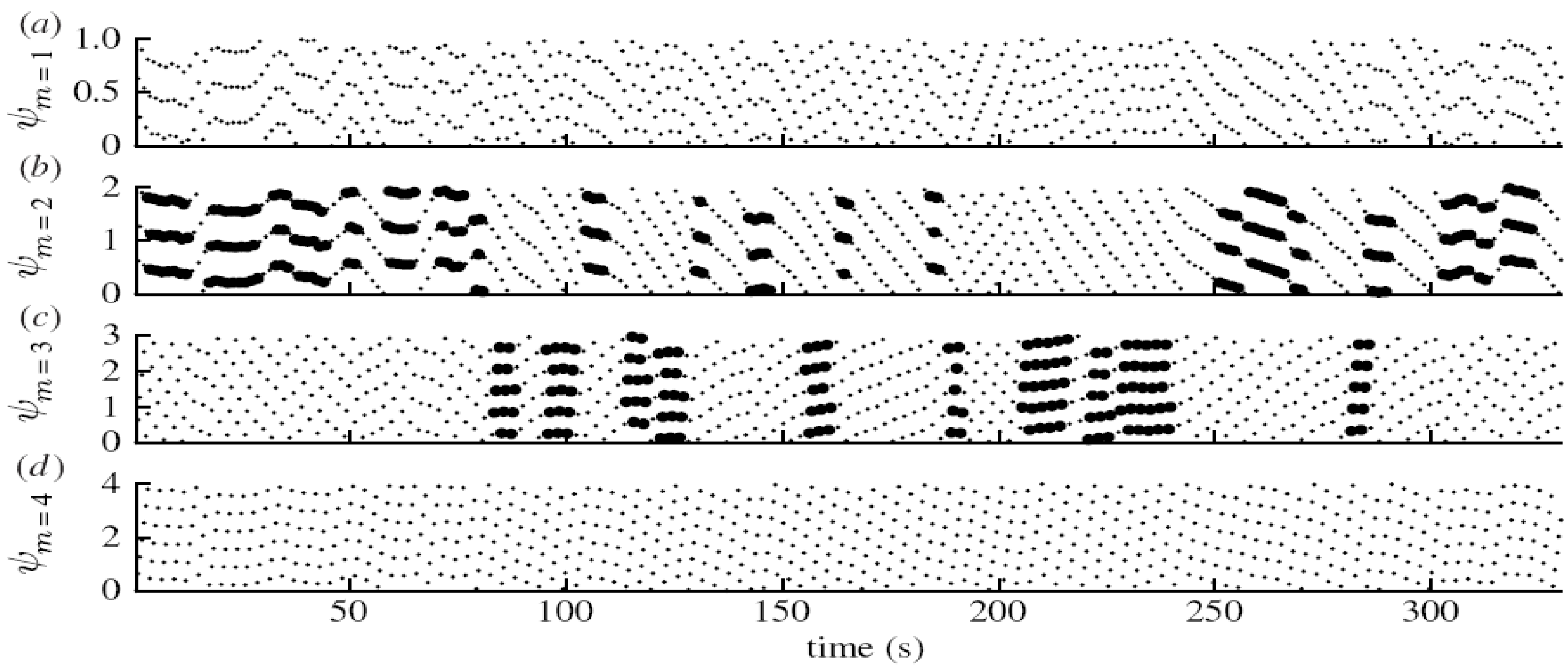
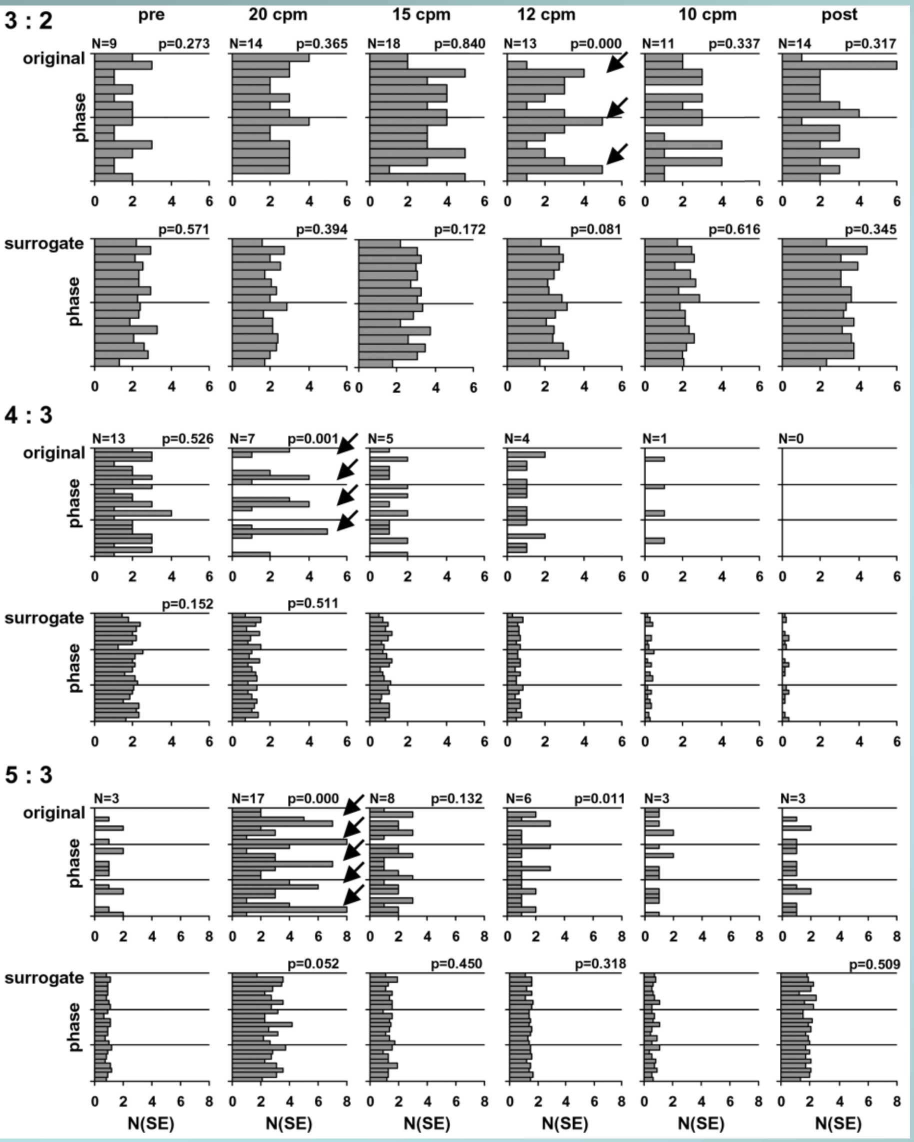


Figure 2. (*a–d*) Synchrograms of the data shown in figure 1 for maternal cycles $m=1, \ldots, 4$. The detected epochs of synchronization (see text) are marked by bold points. The synchronization ratio (*b*) $n:m=3:2$ and (*c*) $n:m=5:3$ were found. The other ratios ($1:1$, $2:1$, $5:2$, $4:3$, $5:4$ and $7:4$) were not detected. For instance, detected $6:4$ synchronization is not marked because it is twice the $3:2$ ratio.

Influence of maternal paced breathing on the connections of the fetal-maternal cardiac systems



Distribution of the synchronization epochs (SE) over the maternal beat phases in the original and surrogate data with respect to the *n*:*m* combinations 3:2 (top), 4:3 (middle) and 5:3 (bottom) in the different respiratory conditions. For the original data, the number of SE found is given at the top left of each graph. As there were 20 surrogate data sets for each original, the number of SE found in the surrogate data was divided by 20 for comparability. The arrows indicate clear phase preferences. *p*-values are given for histograms containing at least 6 SE. (pre, post: data sets of spontaneous breathing prior to and following controlled breathing.)

Special test statistics: twin surrogates

van Leeuwen, Romano, Thiel, Wessel, Kurths, PNAS 106, 13661 (2009) (+ commentary)

Universality in the synchronization of weighted random networks

Our intention:

Include the influence of weighted coupling for complete synchronization

Motter, Zhou, Kurths: Phys. Rev. E 71, 016116 (2005)
Europhys. Lett. 69, 334 (2005)
Phys. Rev. Lett. 96, 034101 (2006)

Our papers on synchronization in complex networks

Europhys. Lett. 69, 334 (2005)
Phys. Rev. E 71, 016116 (2005)
CHAOS 16, 015104 (2006)
Physica D 224, 202 (2006)
Physica A 361, 24 (2006)
Phys. Rev. E 74, 016102 (2006)
Phys: Rev. Lett. 96, 034101 (2006)
Phys. Rev. Lett. 96, 164102 (2006)
Phys. Rev. Lett. 96, 208103 (2006)
Phys. Rev. Lett. 97, 238103 (2006)
Phys. Rev. E 76, 036211 (2007)
Phys. Rev. E 76, 046204 (2007)
Phys. Rev. E 76, 027203 (2007)
New J. Physics 9, 178 (2007)

Phys. Rev. Lett. 98, 108101 (2007)
Phys. Reports 469, 93 (2008)
J. Phys. A 41, 224006 (2008)
Phys. Rev. E 77, 016106 (2008)
Phys. Rev. E 77, 026205 (2008)
Phys. Rev. E 77, 027101 (2008)
CHAOS 18, 023102 (2008)
Europ. J. Phys. B 69, 45 (2009)
Physica A 388, 2987 (2009)
Europhys. Lett. 85, 28002 (2009)
CHAOS 19, 013105 (2009)
Phys. Rev. E 80, 066121 (2009)
Phys. Rev. Lett. 104, 038701 (2010)
Frontiers Neuroinform. 4, 1 (2010)

Physics Reports 469 (2008) 93–153

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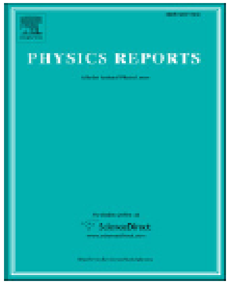

Synchronization in complex networks

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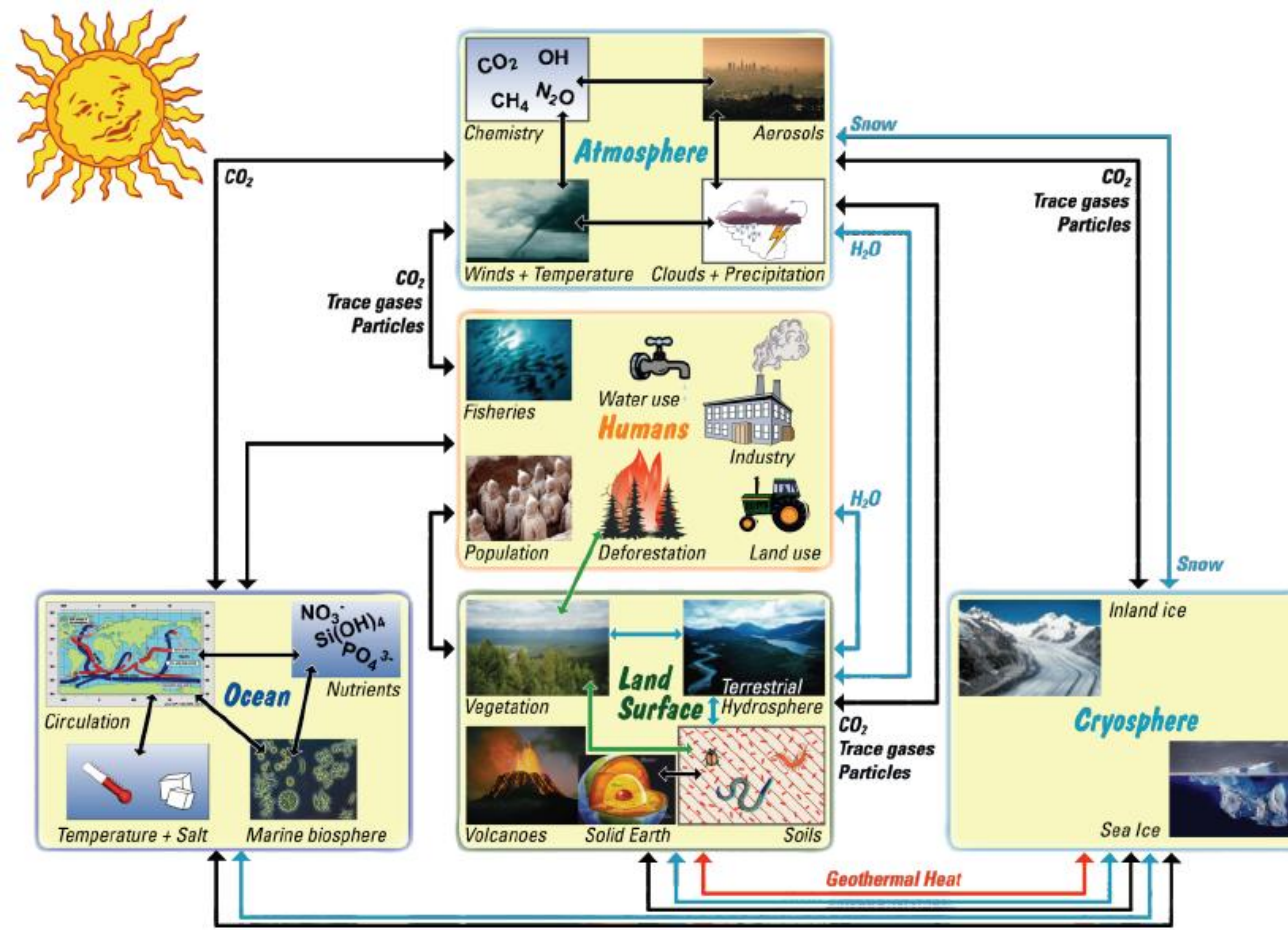
System Earth



Atmosphere
CO2 OH CH4 N2O
Chemistry
Aerosols
Winds + Temperature
Clouds + Precipitation
Snow
CO2
CO2
Trace gases
Particles
H2O
CO2
Trace gases
Particles
Humans
Fisheries
Water use
Industry
Population
Deforestation
Land use
H2O
Snow
Ocean
NO3- Si(OH)4 PO4 3-
Circulation
Nutrients
Temperature + Salt
Marine biosphere
Land Surface
Vegetation
Terrestrial Hydrosphere
Volcanoes
Solid Earth
Soils
CO2
Trace gases
Particles
Cryosphere
Inland ice
Sea Ice
Geothermal Heat

Network Reconstruction from a continuous dynamic system (structure vs. functionality)

New (inverse) problems arise!
(Is there a backbone underlying the climate system?)

Motivation

Envision earth system and its subcomponents (climate system, human realm, ...) as a hierarchical complex network of dynamical systems.

Idea

Apply recent theory of complex networks to the climate system.

P I K

Goals

- Gain novel insights into relation of structure and dynamics (global change).
- Develop new tools for climate data analysis.

Assumptions

- There exists an underlying network structure.
- Correlations in vertex dynamics reveal network topology.

Identifications

- Grid points $\longrightarrow$ vertices.
- Significantly and strongly interrelated pairs of time series $\longrightarrow$ edges.

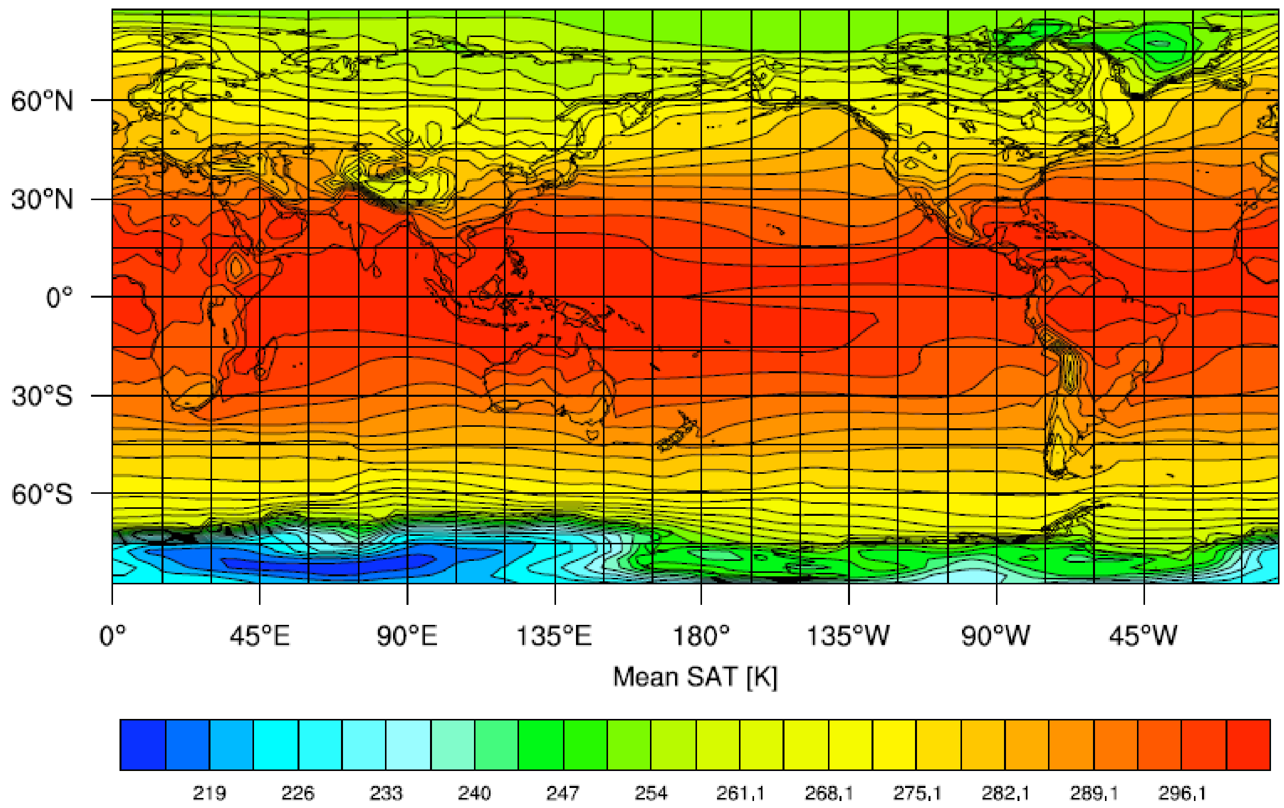
Data Basis: Reanalysis Data

Table: Properties of global NCEP/NCAR surface air temperature data set.

	NCEP/NCAR reanalysis
Temporal coverage	Jan 1948–Dec 2007
Number of samples [months]	720
Lat. resolution [°]	2.5
Lon. Resolution [°]	2.5
Number of vertices	10,224

P I K

Mean SAT field



Network Reconstruction

- Interactions among activity at the nodes
  - linear: cross-correlations

- General: Mutual information

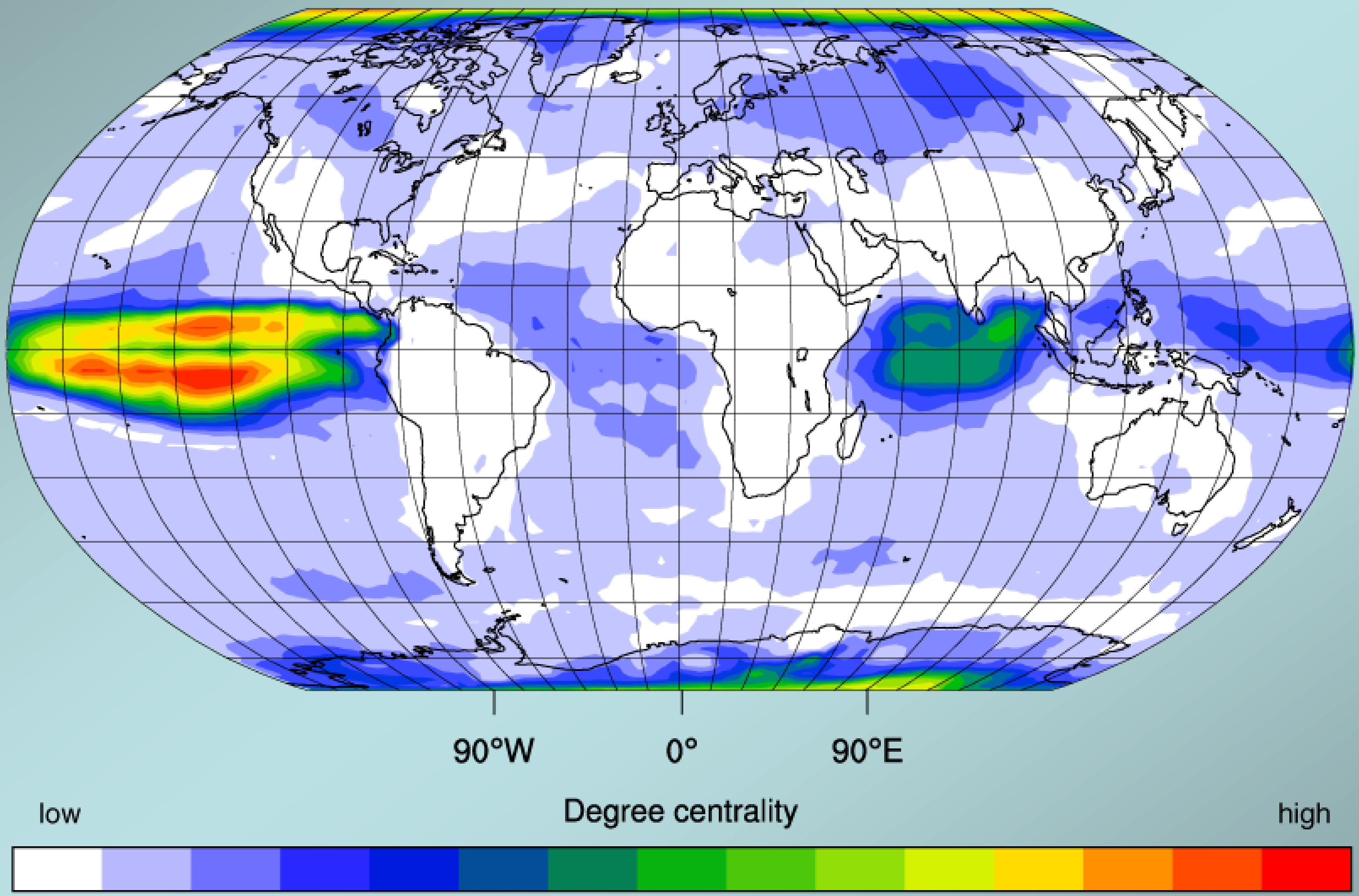
$$MI_{ij} = \sum_{\mu\nu} p_{ij}(\mu,\nu) \log \frac{p_{ij}(\mu,\nu)}{p_i(\mu) p_j(\nu)}$$

Obtaining the adjacency matrix

- Obtain adjacency matrix A_{ij} from mutual information matrix M_{ij} by thresholding,

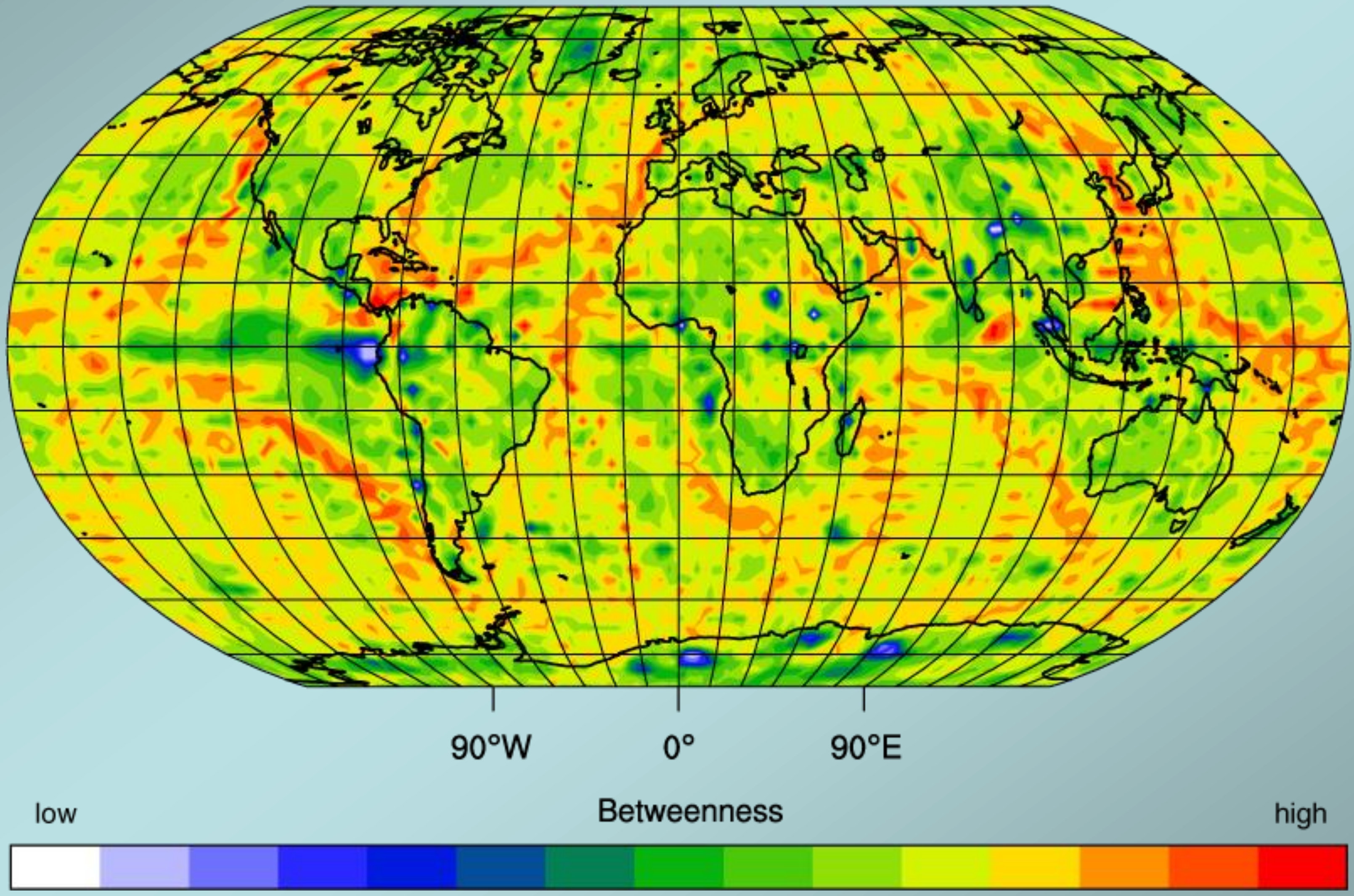
$$A_{ij} = \Theta\left(M_{ij} - \tau\right) .$$

- Significance tests for threshold selection.

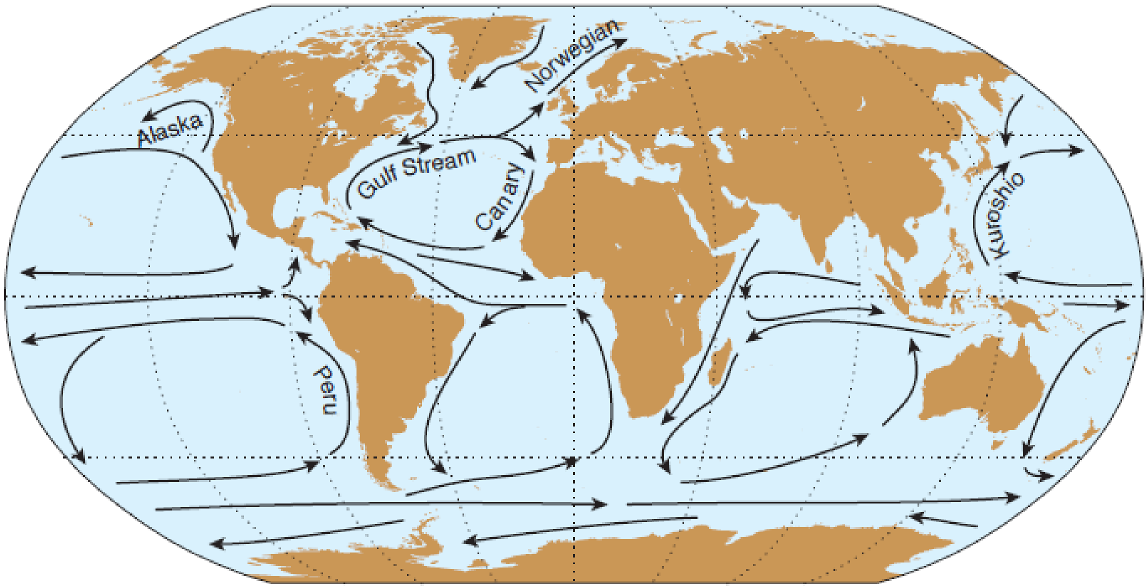


90°W
0°
90°E
low
Degree centrality
high

Betweennes Centrality

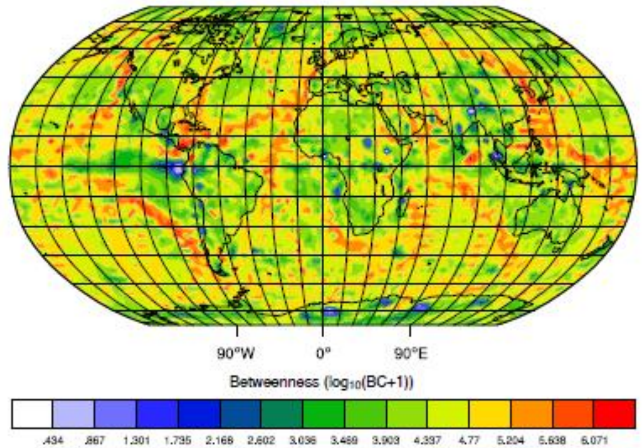


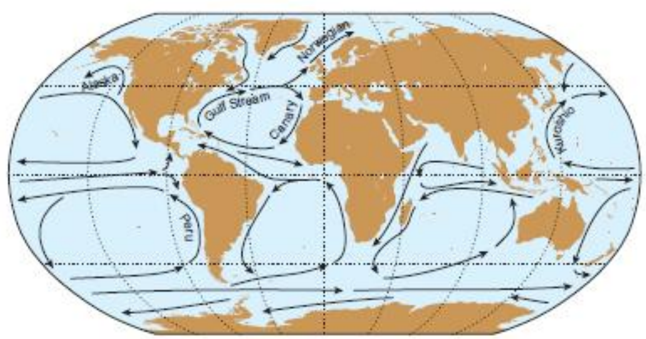
Global surface ocean currents



Donges et al., Europhys Lett **87**, 48007 (2009)

Backbone structures → surface ocean currents?





Donges et al., Europhys Lett **87**, 48007 (2009)

Separation in time:

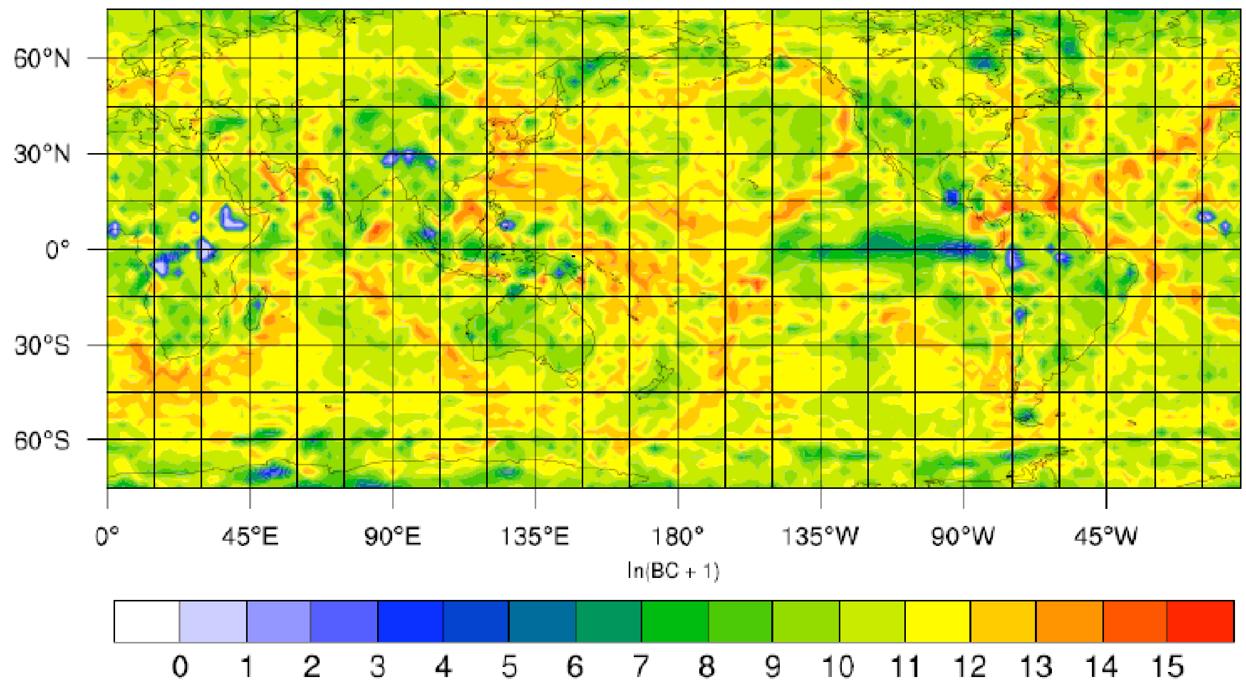
Part I: 1948 – 1974

(stationary)

Part II: 1975 – 2008

(climate change)

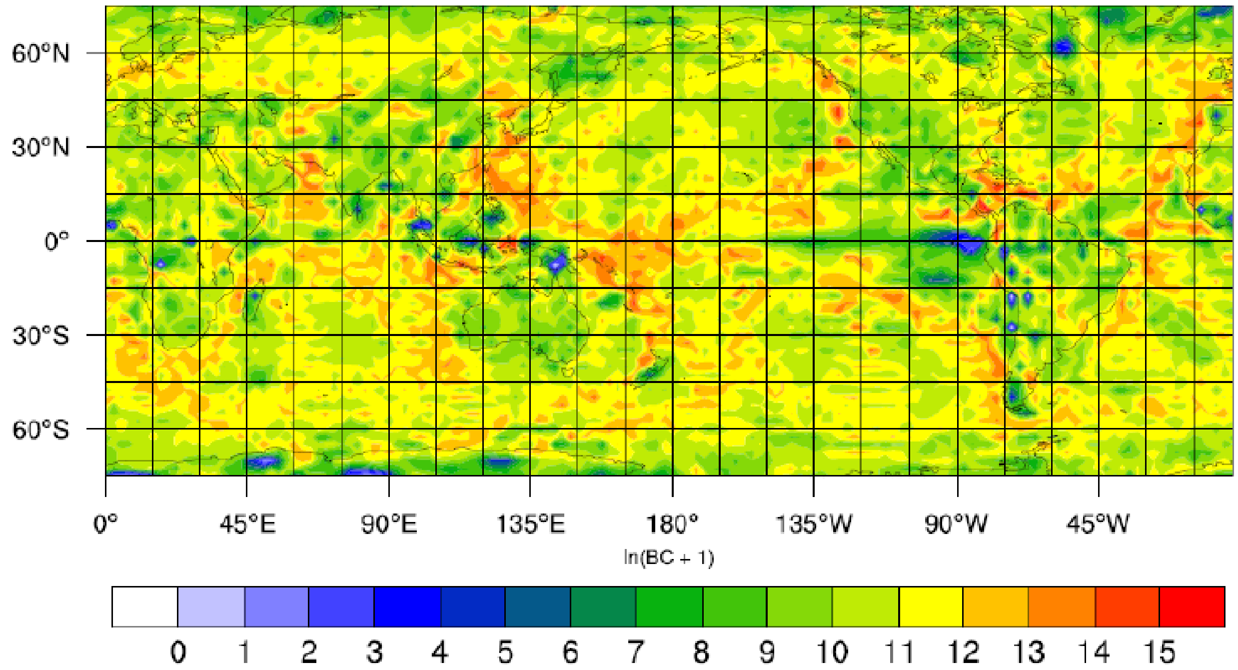
Betweenness field 1948–1978



Map by **Hanna Schultz**

PIK

Betweenness field 1979–2007



Map by **Hanna Schultz**

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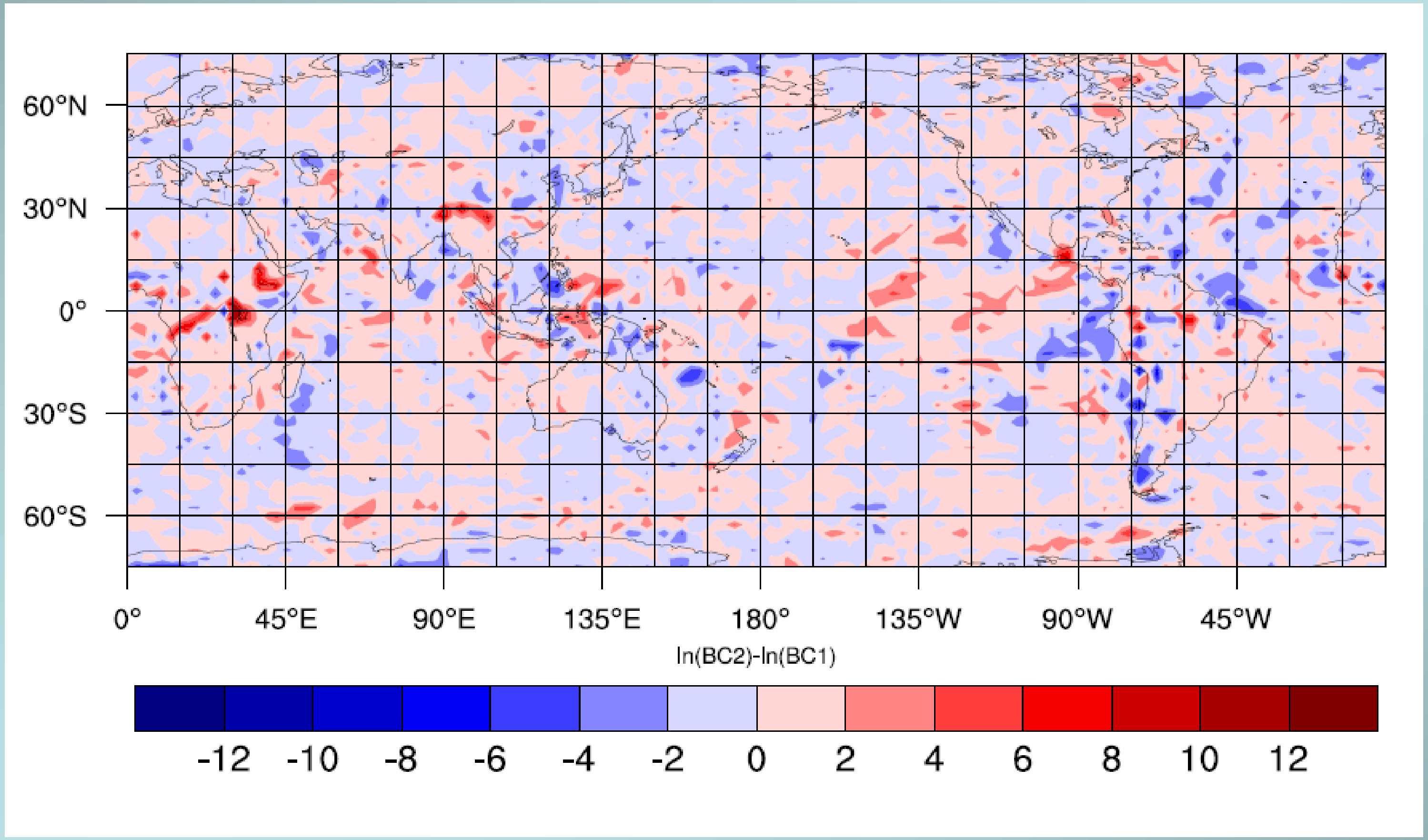
Log BC2 – Log BC1

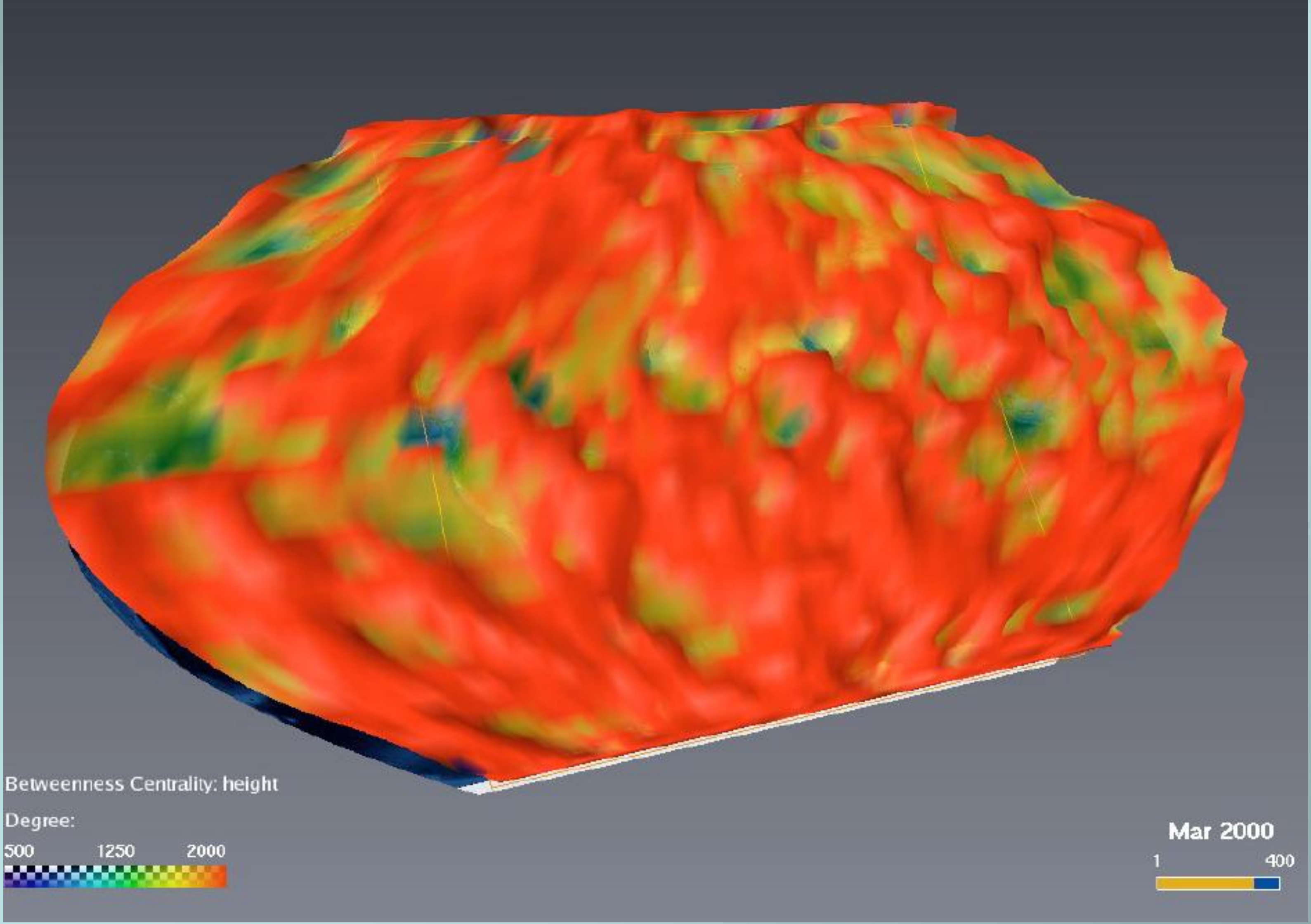


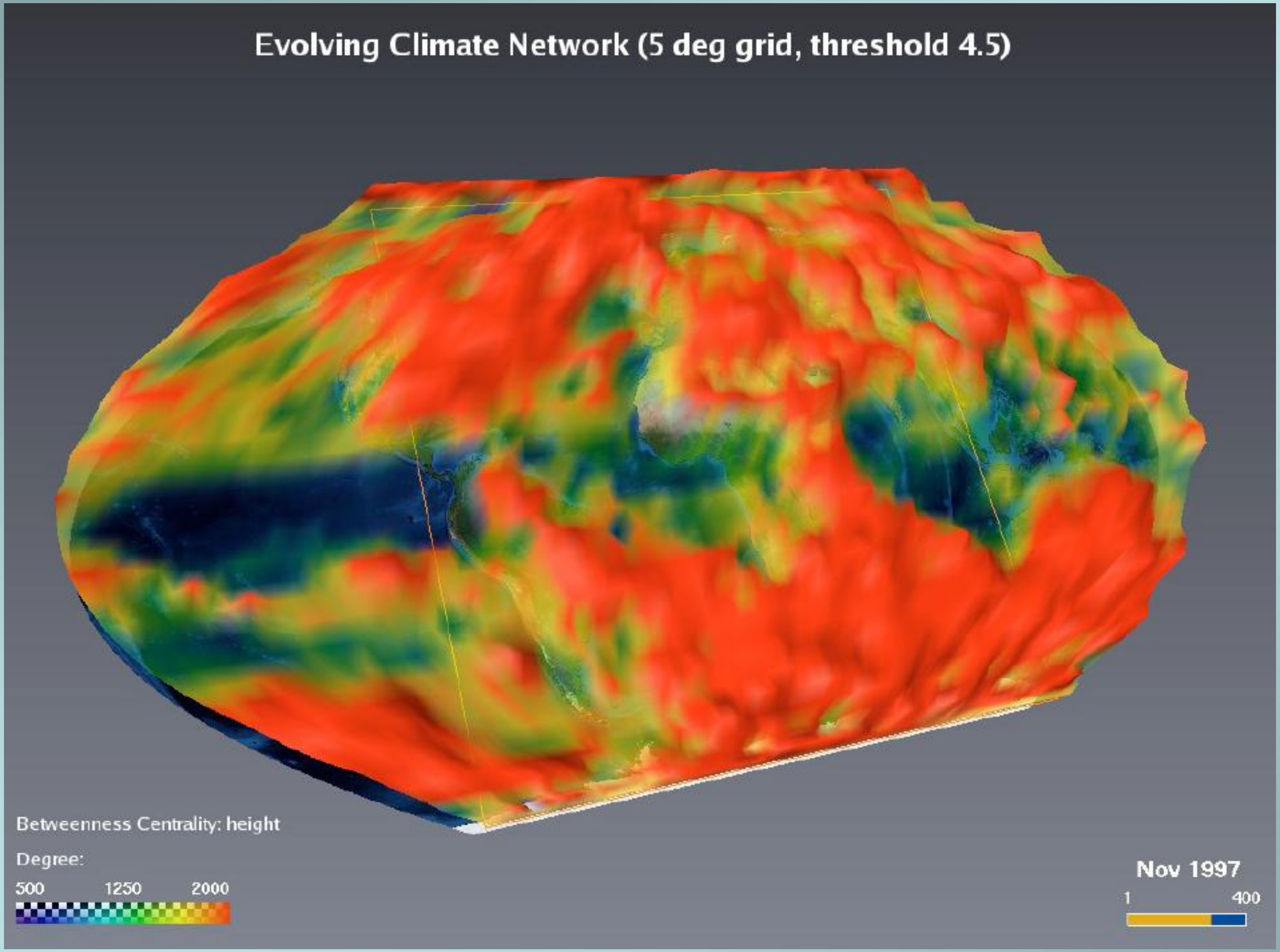
Time evolution

- Daily data
- Sliding windows of one year length
- Betweenness centrality and degree centrality

- Network topology changes due to natural dynamics (internal, external)

Evolving Climate Network (5 deg grid, threshold 4.5)





Strong El Nino event

Our papers on climate networks

Europhys. Lett. 87, 48007 (2009)

Europ. Phys. J. Special Topics,174, 157-179 (2009)

Concept of Recurrence

Recurrence

Περιχωρεσιζ – perichoresis (Anaxagoras)

Recurrence plots

- **Recurrence plot**

  $R(i, j) = \Theta(\varepsilon - |x(i) - x(j)|)$

Θ – Heaviside function

ε – threshold for neighborhood (recurrence to it) - (Eckmann et al., 1987)

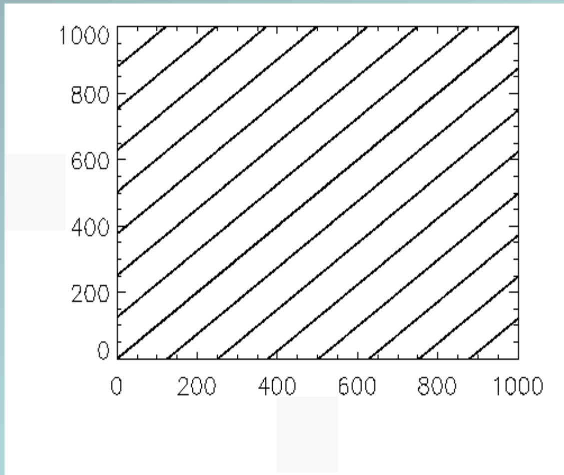
Generalization for Data Analysis:

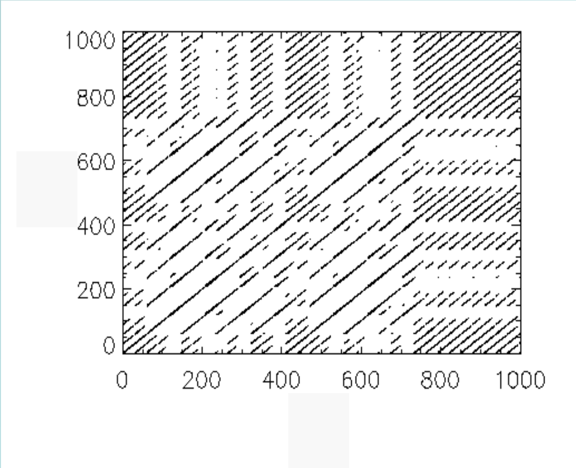
Statistical properties of all side diagonals and vertical elements

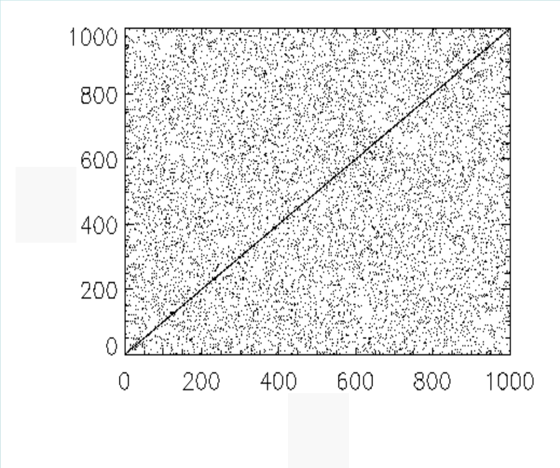
Sine

Rössler oscillator

white noise







Predictability
Long diagonals

Unpredictability
Short diagonals

Distribution of the Diagonals

$$P_\varepsilon(l) \approx \varepsilon^{D_2} \exp(-\tau K_2 l)$$

The following parameters can be estimated by means of RPs (Thiel, Romano, Kurths, CHAOS, 2004):

Correlation Entropy:

$$\hat{K}_2(\varepsilon, l) = \frac{1}{l\tau} \ln\left(\frac{1}{N^2} \sum_{s,t=1}^{N} \prod_{m=0}^{l-1} R_{t+m,s+m} \right)$$

Correlation Dimension:

$$\hat{D}_2(\varepsilon, l) = \ln\left(\frac{P_\varepsilon(l)}{P_{\varepsilon+\Delta\varepsilon}(l)} \right) \Big/ \left(\frac{\varepsilon}{\varepsilon + \Delta\varepsilon} \right)$$

Mutual Information:

$$\hat{I}_2(\varepsilon, \tau) = -2 \ln\left[\frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \right] + \ln\left[\frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} R_{i+\tau,j+\tau} \right]$$

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Physics Reports 438 (2007) 237–329

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Recurrence plots for the analysis of complex systems

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editor: I. Procaccia

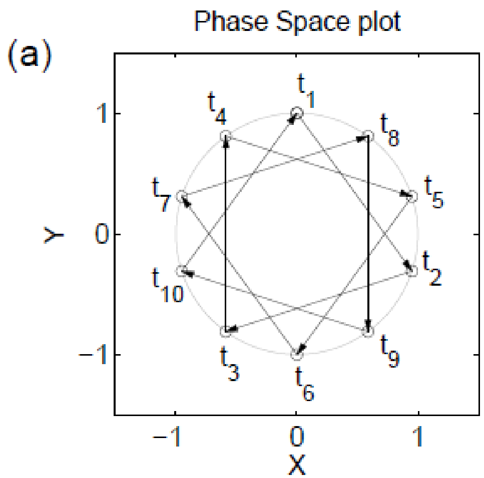
Complex network approach for recurrence analysis – Recurrence Networks

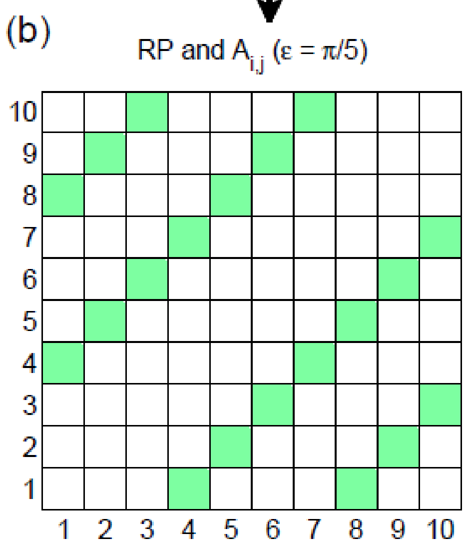
- Interprete a recurrence matrix obtained from a dynamical system as adjacency matrix and refer to a network with complex topology

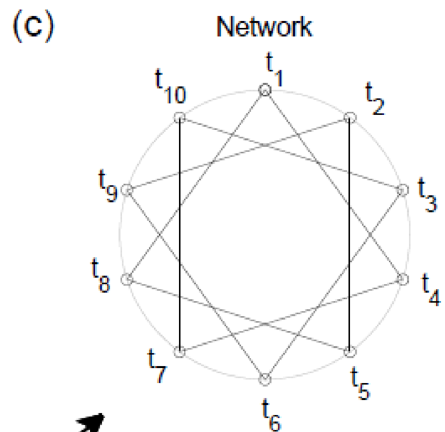
$$A_{i,j} = R_{i,j} - \delta_{i,j}$$

- Elements are
- Use of typical complex networks parameters: degree, betweennes, clustering

Transform of a periodic trajectory to a network







(d) Shortest Path Length list: $l_{i,j}$

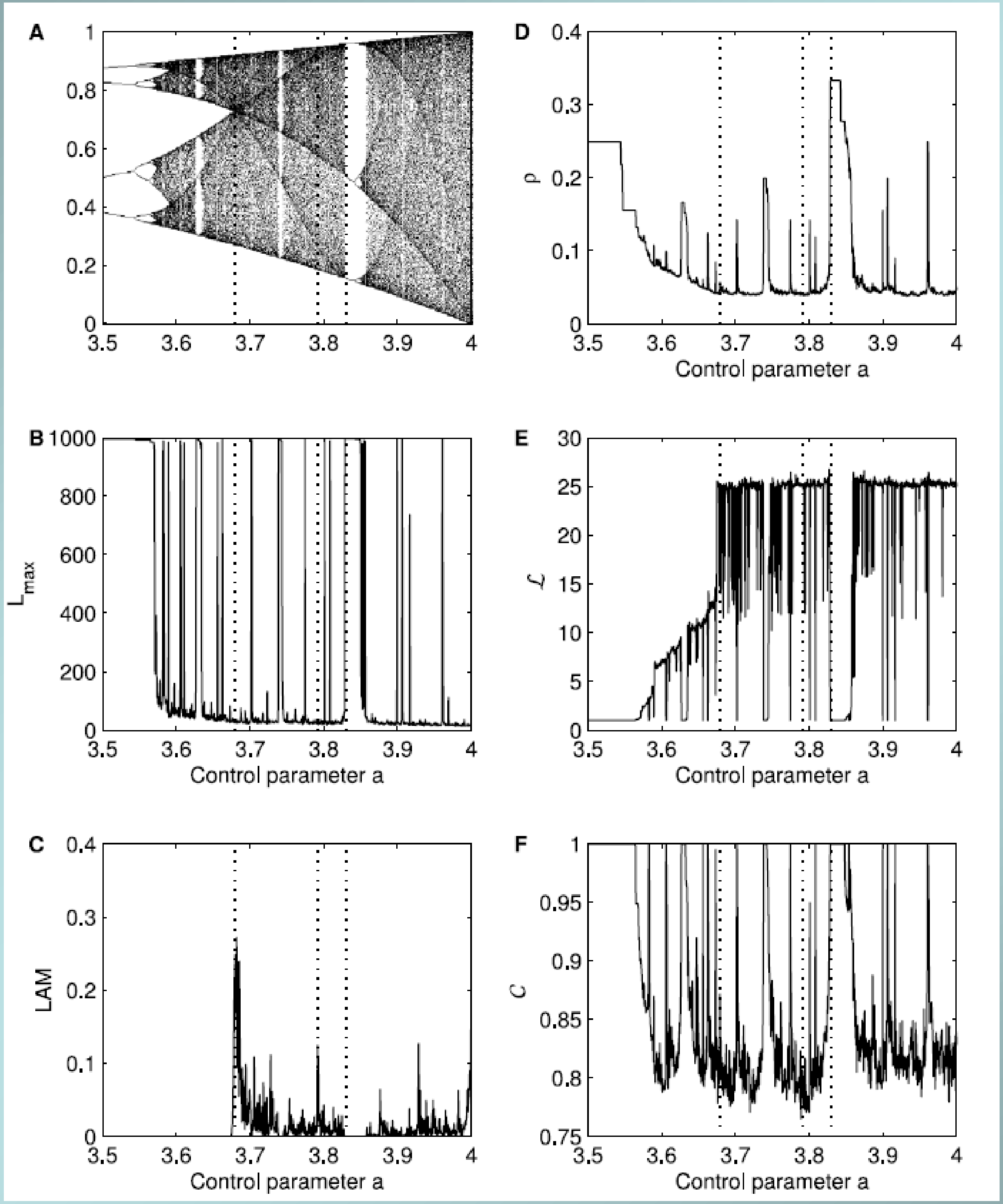
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
t_1		3	4	1	2	5	2	1	4	3
t_2	3		3	4	1	2	5	2	1	4
t_3	4	3		3	4	1	2	5	2	1
t_4	1	4	3		3	4	1	2	5	2
t_5	2	1	4	3		3	4	1	2	5
t_6	5	2	1	4	3		3	4	1	2
t_7	2	5	2	1	4	3		3	4	1
t_8	1	2	5	2	1	4	3		3	4
t_9	4	1	2	5	2	1	4	3		3
t_{10}	3	4	1	2	5	2	1	4	3	

$x(t) = \sin(6\pi \cdot 0.1t),\ y(t) = \cos(6\pi \cdot 0.1t)$

$\epsilon = \frac{\pi}{5}$

Typical network parameters

- Degree centrality $$k_v = \sum_{i=1}^{N} A_{v,i}$$
- Link density $$\rho = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} A_{i,j}$$
- Clustering Coefficient $$C_v = \frac{\sum_{i,j=1}^{N} A_{v,i} A_{i,j} A_{j,v}}{k_v (k_v - 1)}$$
- Average path length $$\mathcal{L} = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} d_{i,j}$$



A
B
C
D
E
F
L_max
LAM
ρ
ℒ
C
Control parameter a

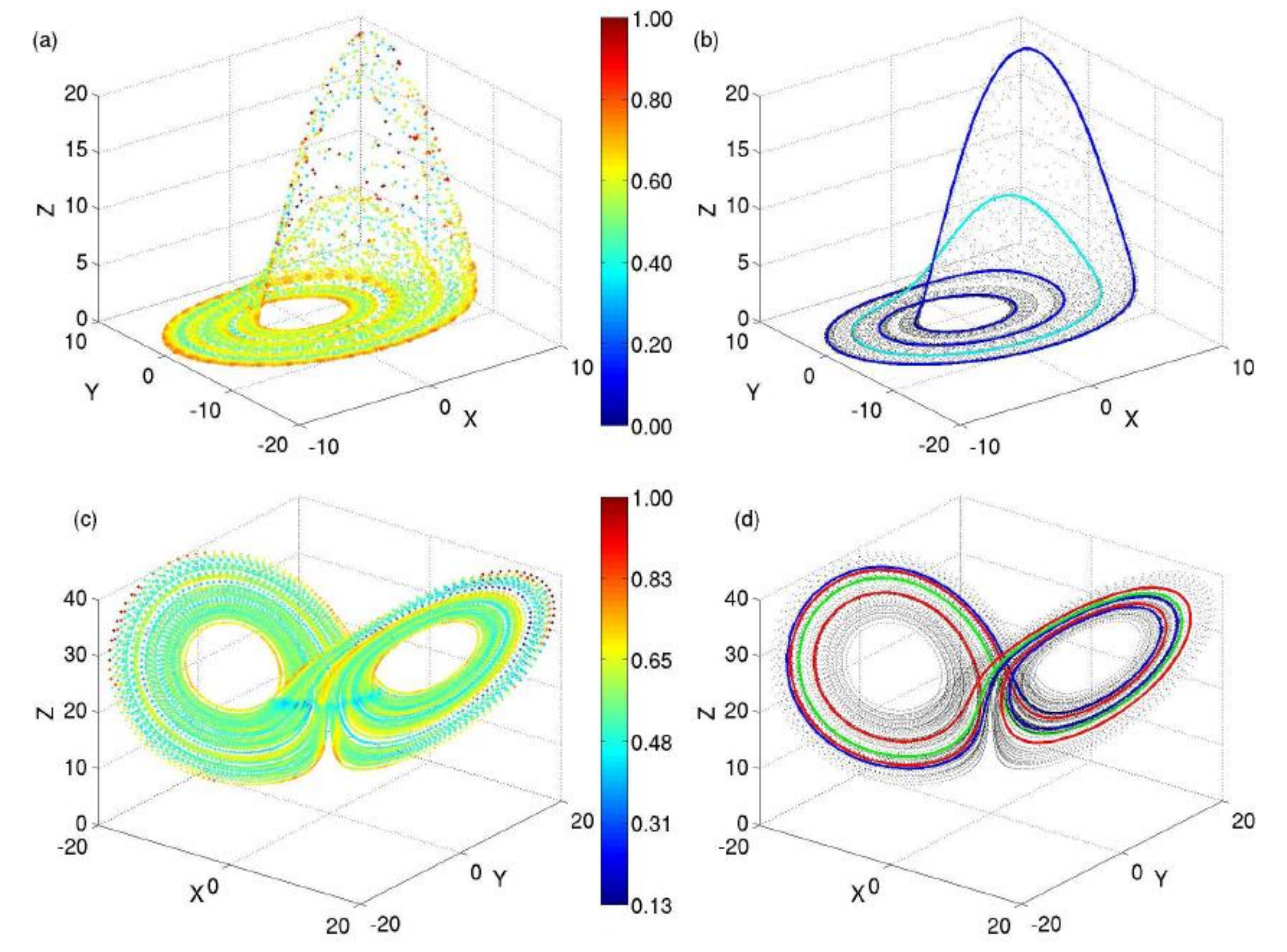


Figure 10. (a) Colour-coded representation of the local clustering coefficients $\mathcal{C}_v$ for the Rössler system ($N = 10,000$) in its phase space. (b) Several locations of the unstable periodic orbits with low periods, obtained using a method based on the windows of parallel lines in the corresponding recurrence plot as described in [9]. The chosen value of ϵ corresponds to a global recurrence rate of $RR = 0.01$. (c),(d) Same as (a),(b) for the Lorenz system ($N = 20,000$).

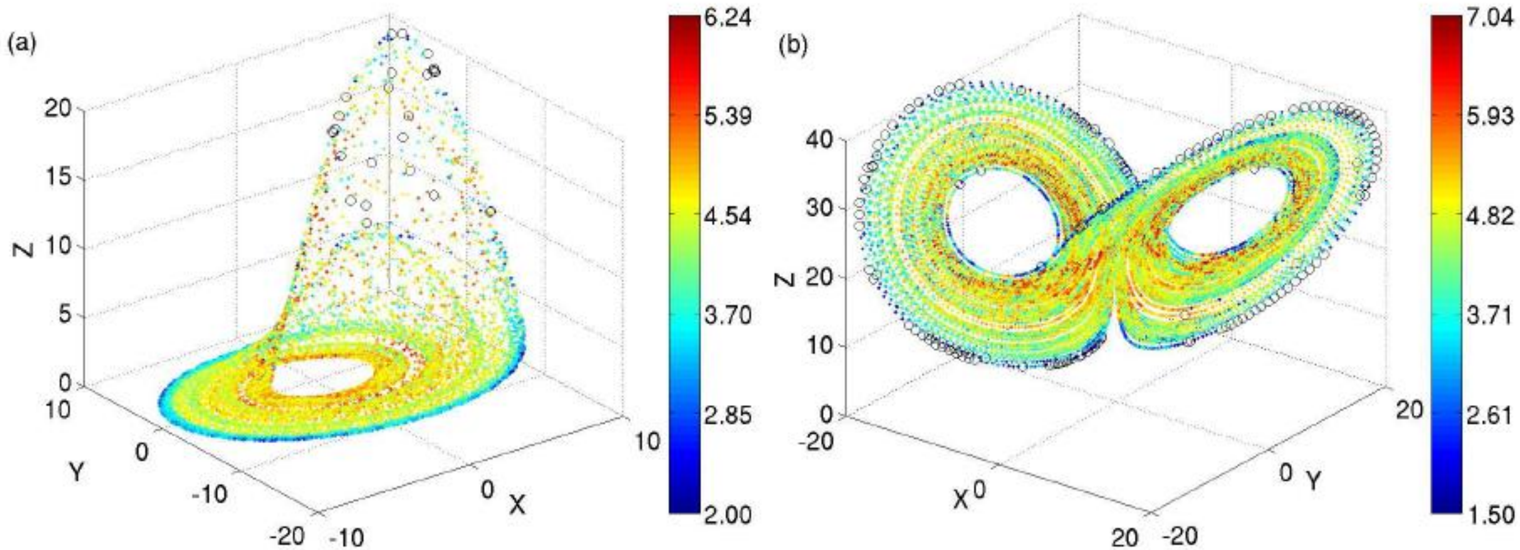


Figure 11. Logarithm of the betweenness centrality b_v for (a) Rössler and (b) Lorenz system (N and ϵ as in Fig. 10). Points shown as circles have betweenness values below the lower limit of the displayed colour scale.

Palaeoclimatic Data

- Marine record from ocean drilling programme (ODP) in the atlantic, site 659
- Marine terrigenous dust measurements

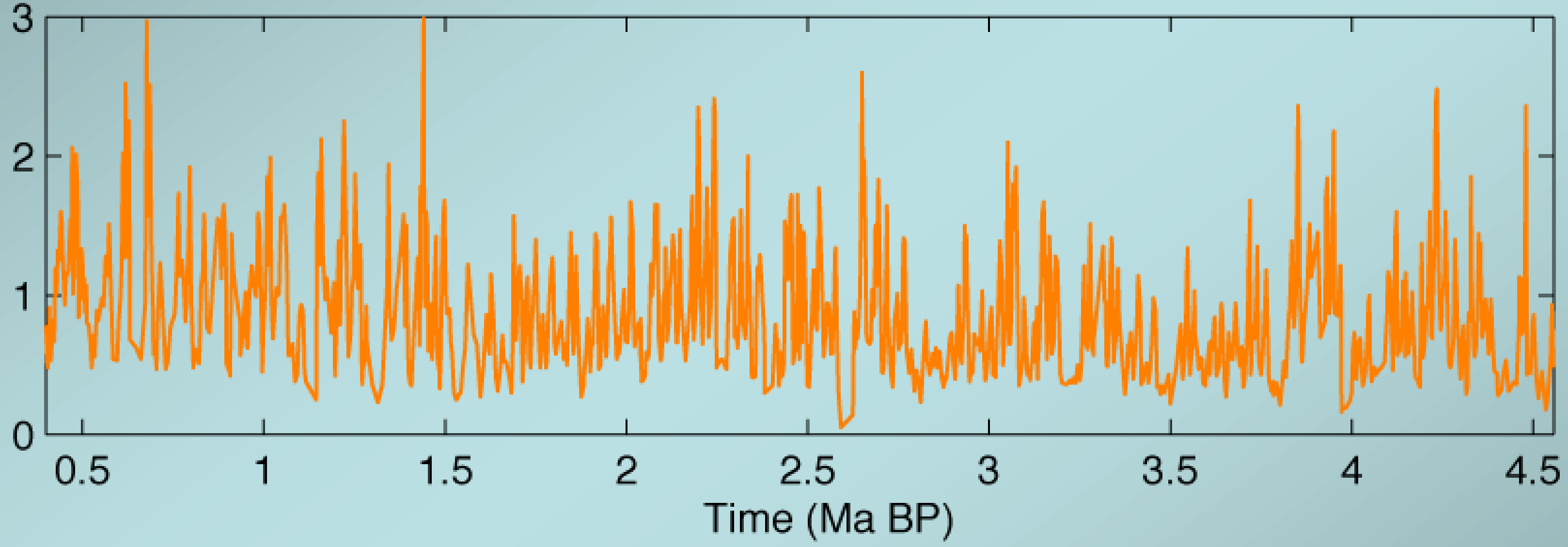
➔epochs of arid continental climate in Africa

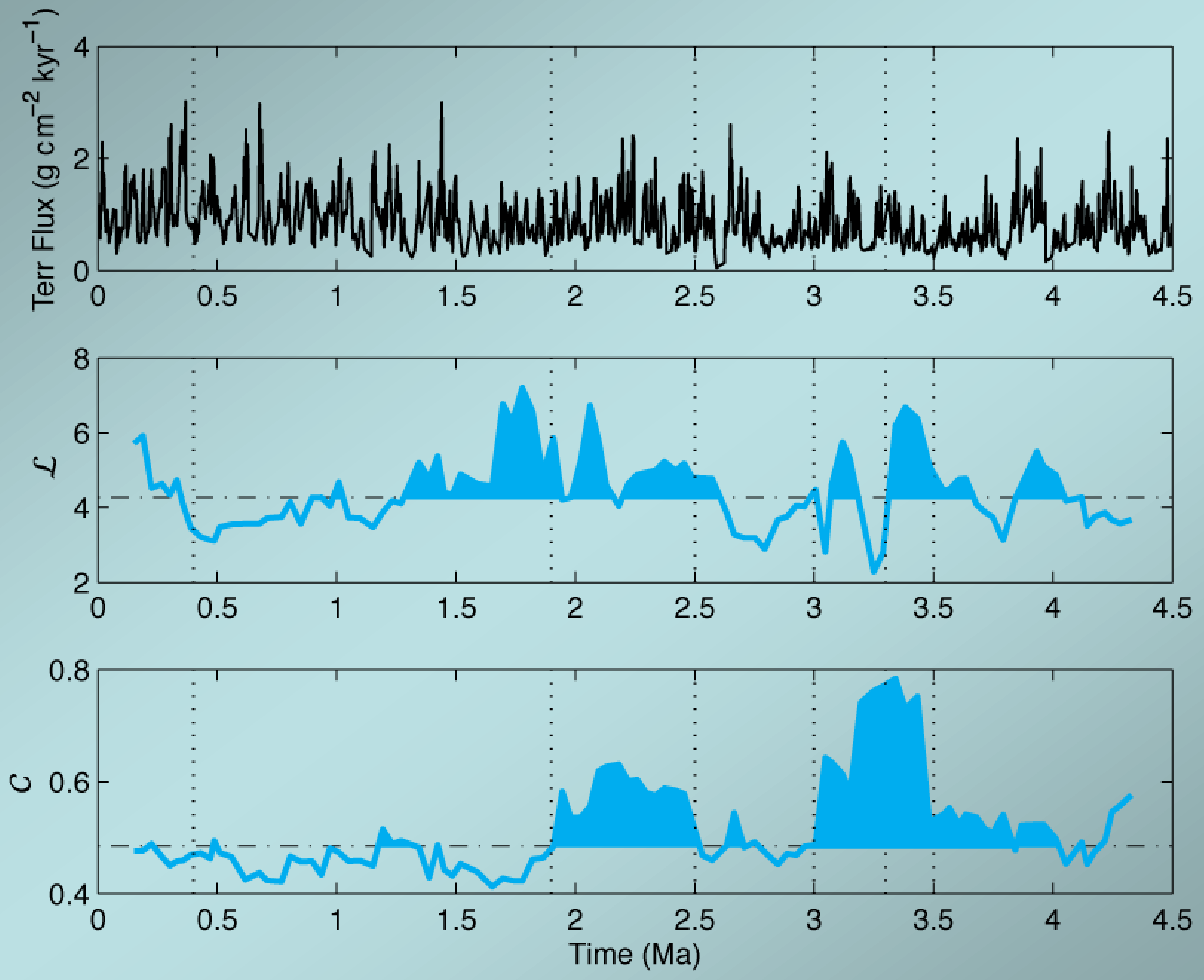
Record covers the last 4.5 Ma,

sampling = 4.1 ka, N = 1240

30° N
15° N
0°
30° W
15° W
0°
ODP 659
AFRICA

Time Series





Terr Flux (g cm^{-2} kyr^{-1})
4
2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\mathcal{L}$
8
6
4
2
$\mathcal{C}$
0.8
0.6
0.4
Time (Ma)

Main Results

- RP shows a homogeneous behaviour interrupted by small bands of sparse structures
- Cluster coeff: signif. Max 3.5-3.0 and 2.5-2.0 (dominant Milankovich cycles 41 ka-world)
- Average path length: signif. Max. 3.5-3.3, ~2.1, 1.9-1.8, ~0.4 and sudden drops at 3.3, 2.0, 1.9 (max here during changes of C – refer to transition periods)

Our papers on recurrence networks

- Phys. Lett. A 373, 4246 (2009)
- Phys. Rev. E 81, 015101R (2010)
- New J. Phys. 12, 033025 (2010)