Large Eddy Simulation Applications to Meteorology

Marcelo Chamecki

Department of Meteorology

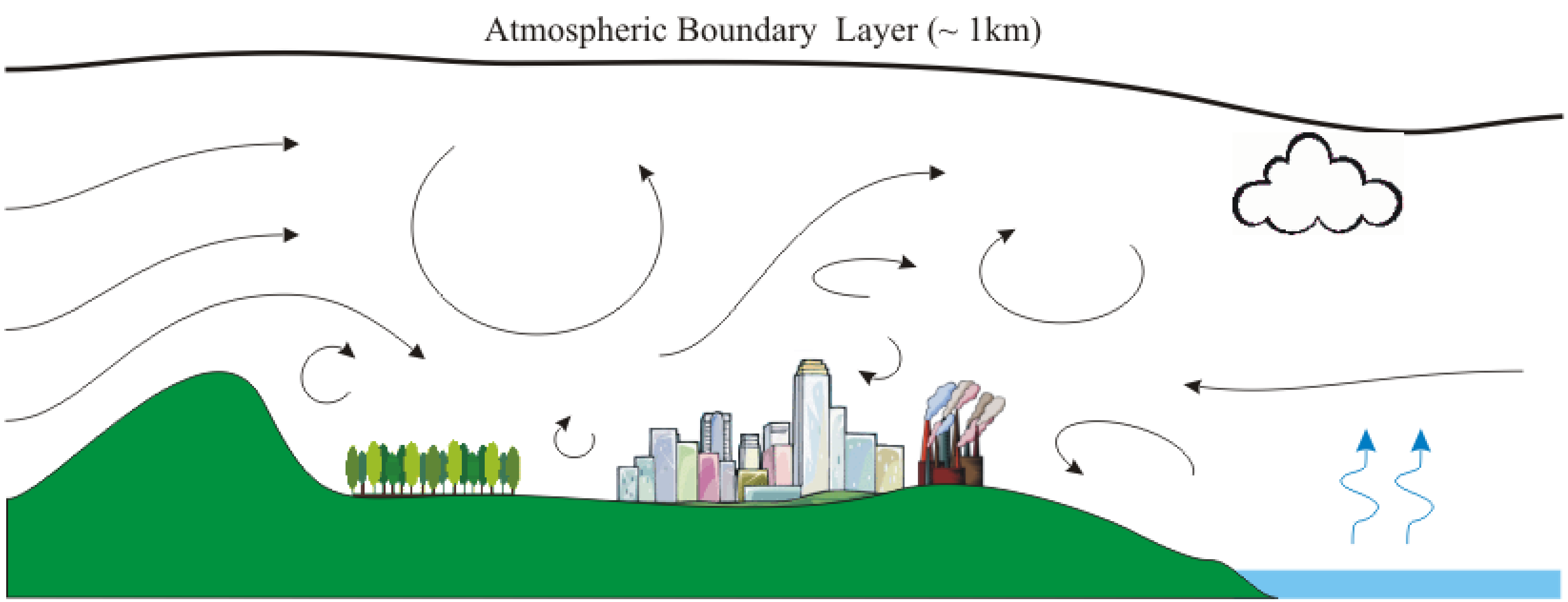
The Pennsylvania State University

Tutorial School on Fluid Dynamics: Topics in Turbulence

May 27th 2010, College Park, MD

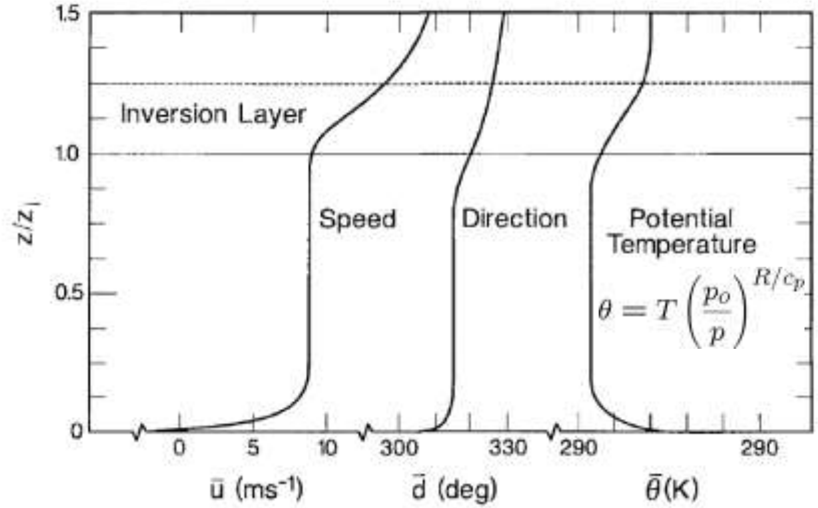
The Atmospheric Boundary Layer (ABL)

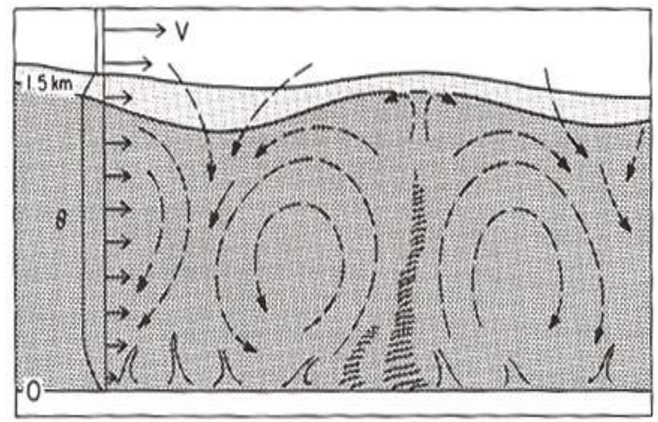
- ABL: lowest region of the atmosphere, directly affected by the Earth's surface
- Interaction with surface: turbulent fluxes of momentum, heat, water vapor, etc.
- The ABL has a strong diurnal cycle due to solar radiation
- First order approximation: very high-Re (~10^7) turbulent channel flow, over rough wall, strongly affected by buoyancy forces



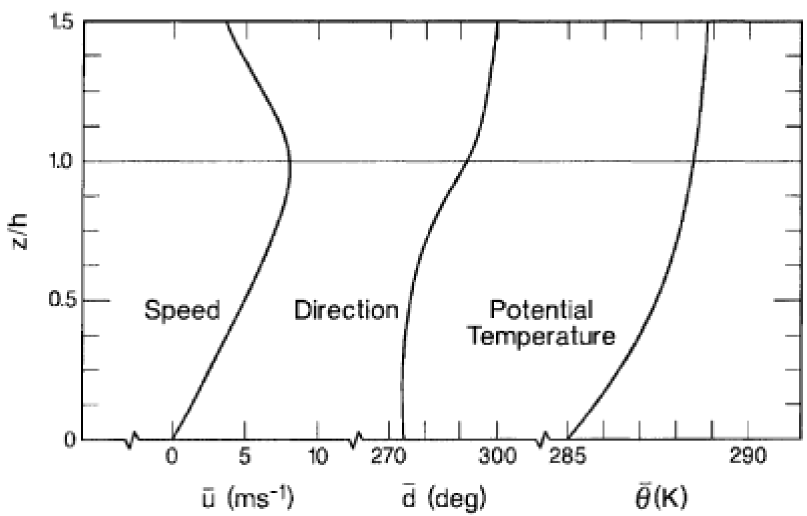
The two canonical states of ABL

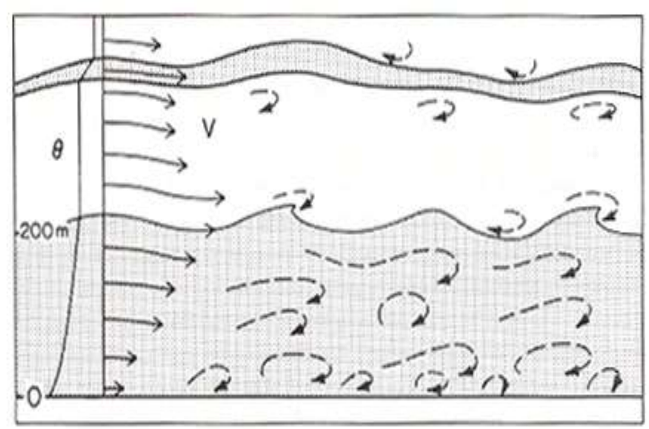
- Daytime ABL (unstable temperature stratification)





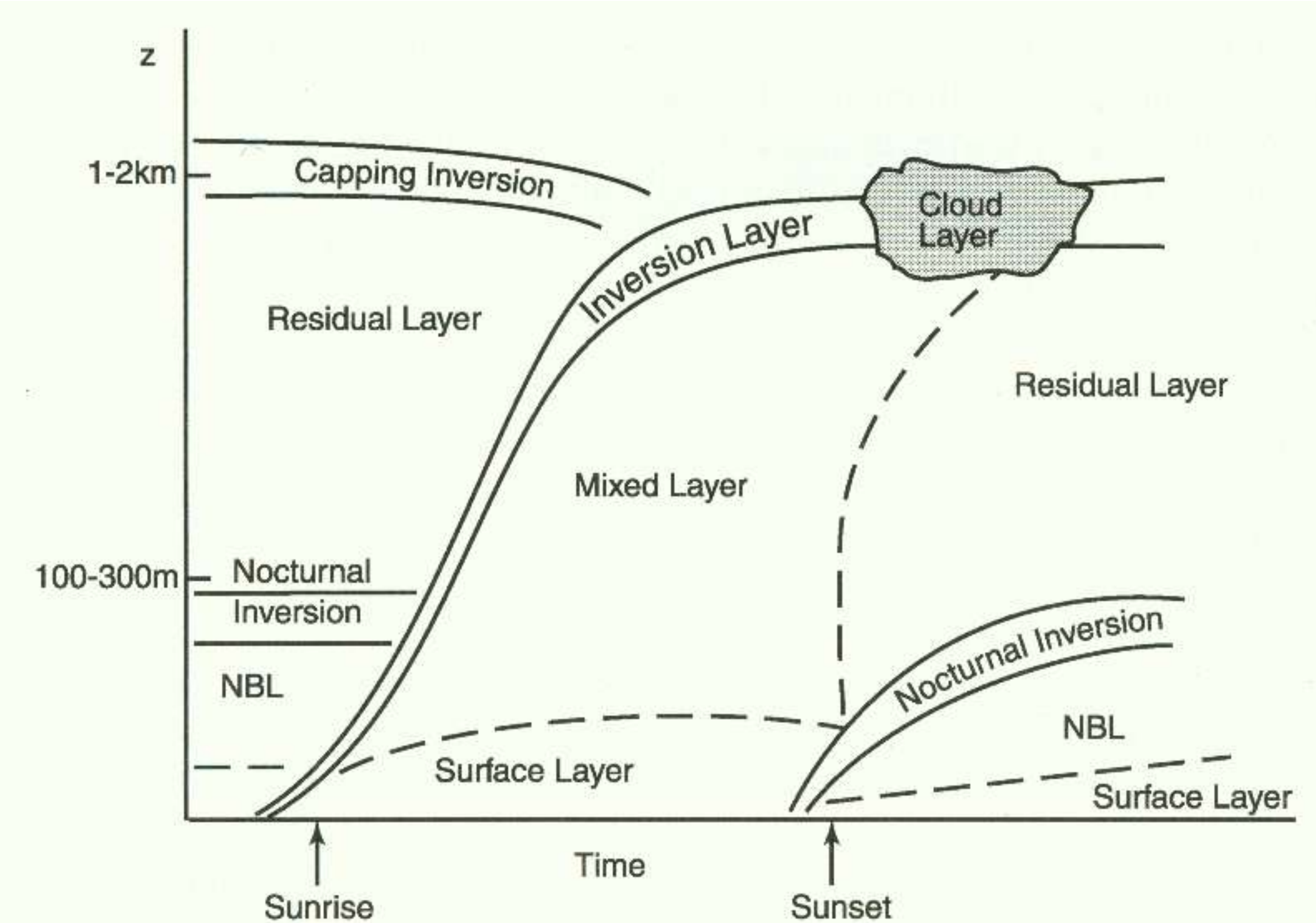
- Nighttime ABL (stable temperature stratification)



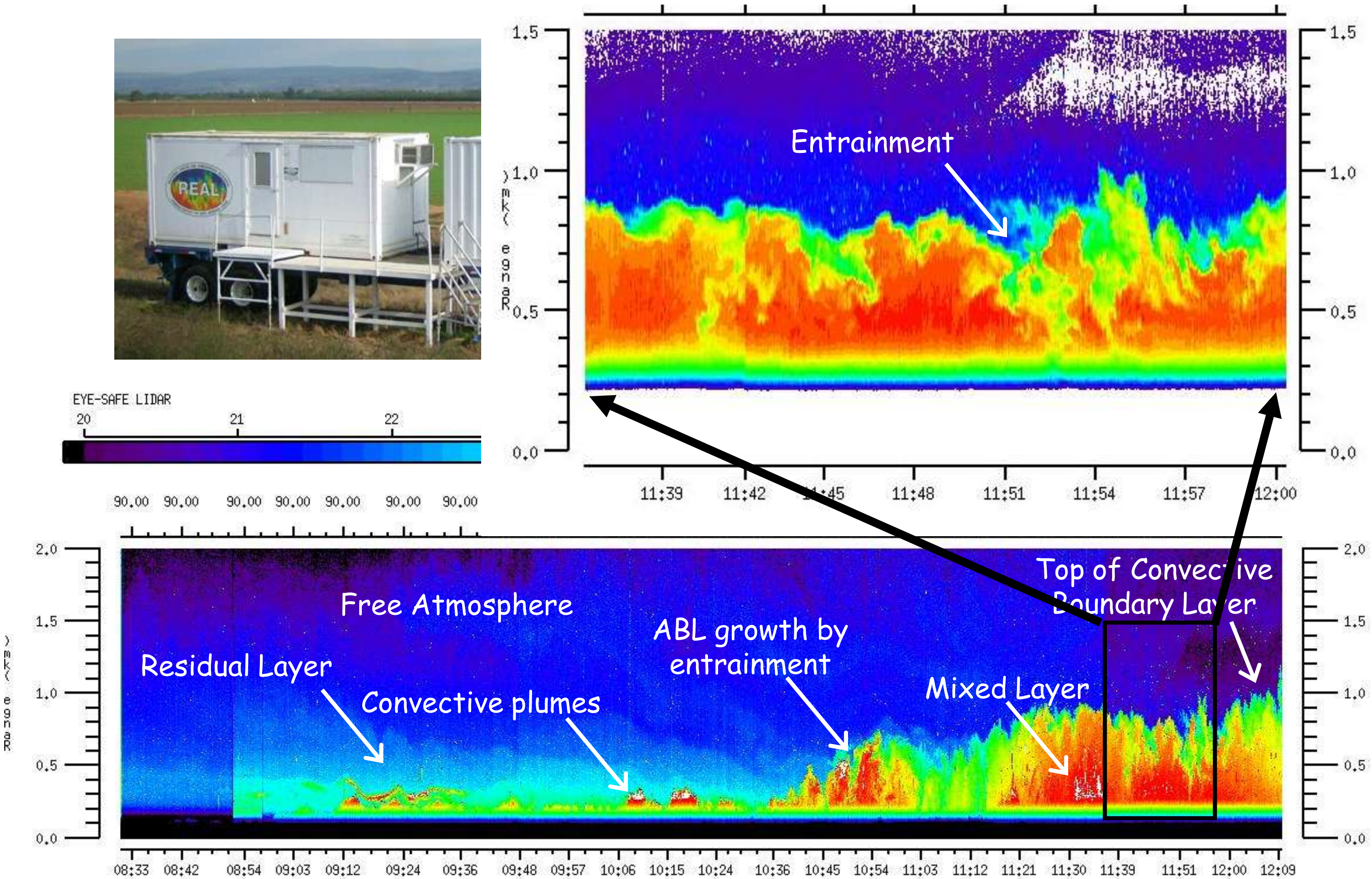


Figures from Kaimal & Finnigan (1994)

Diurnal evolution of the ABL

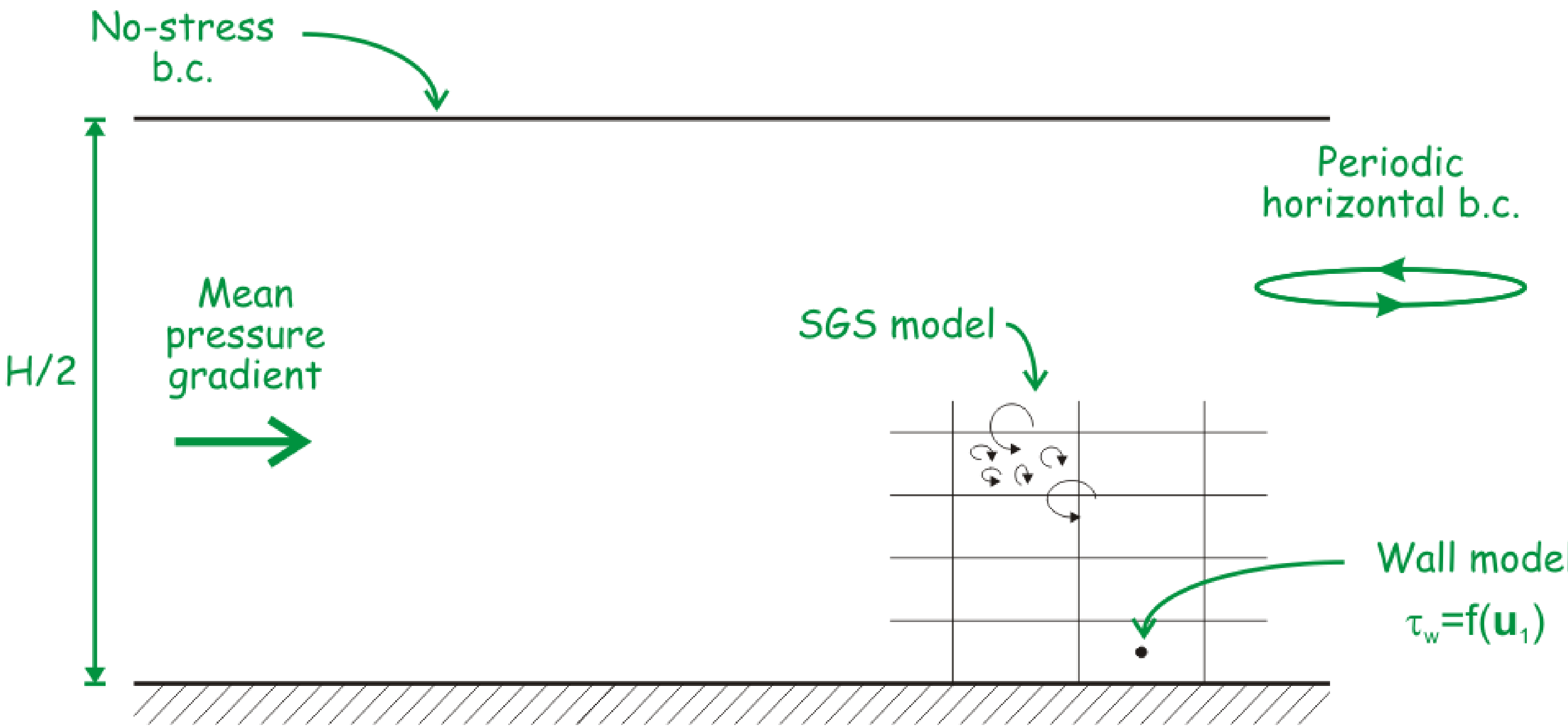


Observations using LIDAR

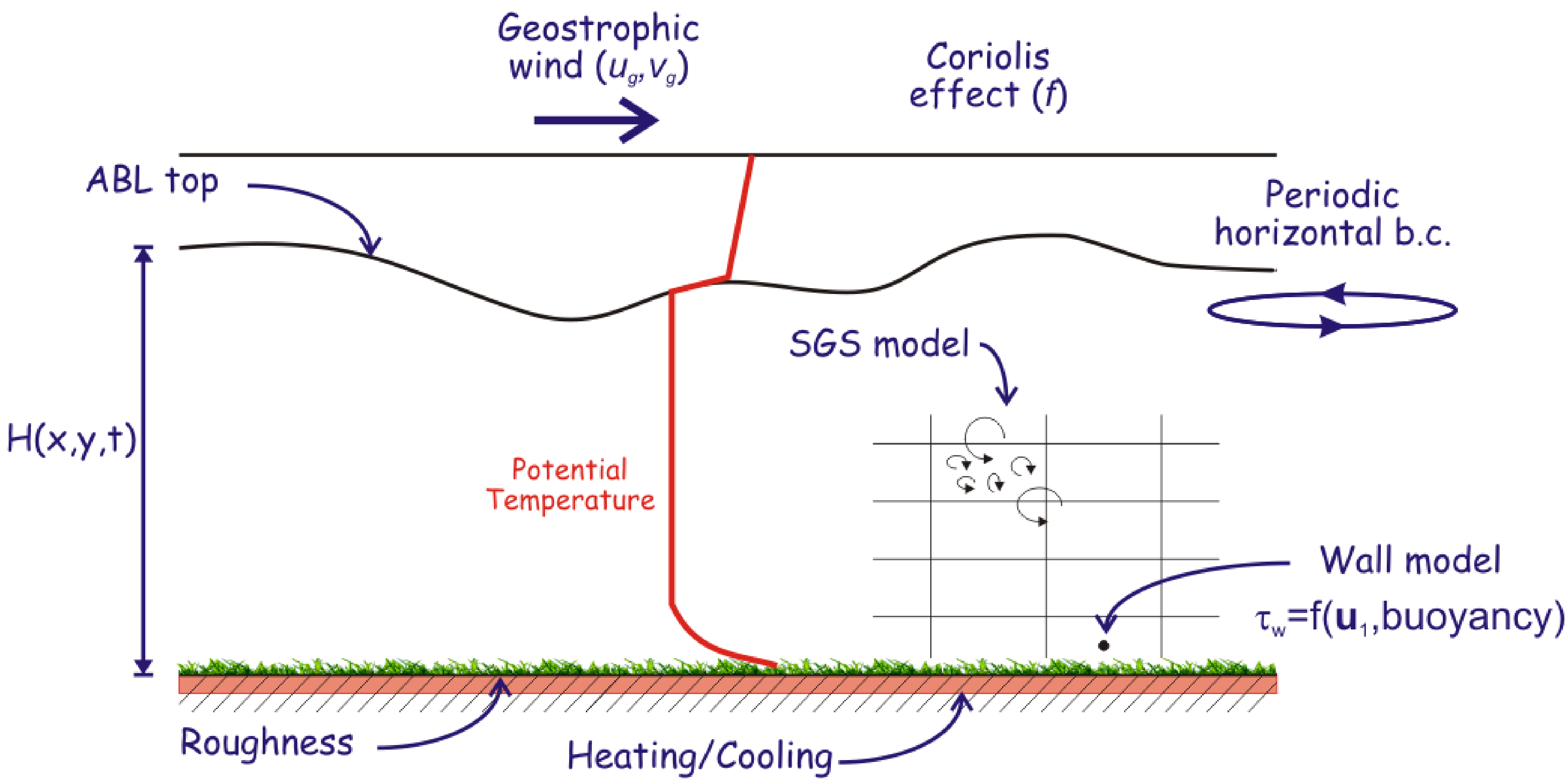


Figures from NCAR website: www.eol.ucar.edu/lidar/real/obs/obs.html

LES: from high-Re channel to ABL flows



LES: from high-Re channel to ABL flows



LES of ABL flows: equation set

- Filtered Navier-Stokes equations (Boussinesq approx.):

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\frac{1}{\rho} \nabla \tilde{p} + \mathbf{g} - \underbrace{\frac{\tilde{\theta}}{\langle \tilde{\theta} \rangle} \mathbf{g}}_{\text{Buoyancy force}} - \underbrace{2\boldsymbol{\Omega} \times \tilde{\mathbf{u}}}_{\text{Coriolis force}} + \nu \nabla^2 \tilde{\mathbf{u}} - \underbrace{\nabla \cdot \boldsymbol{\tau}}_{\text{Subgrid scale (SGS) force}} \qquad \tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\theta} = \frac{\nu}{Pr} \nabla^2 \tilde{\theta} - \underbrace{\nabla \cdot \boldsymbol{\pi}}_{\text{Div of SGS heat flux}} \qquad \pi_i = \widetilde{u_i \theta} - \tilde{u}_i \tilde{\theta}$$

- In this “simplified approach”, the two critical components of a good LES simulation are:

  - wall models

  - SGS models

Boundary conditions – Wall models

- The physical boundary condition:

  - No-slip at solid boundaries $\tilde{\mathrm{u}} = 0$

- Problem for LES of high-Re flows:

  - Cannot resolve sharp gradients!

- The numerical boundary condition:

  - Equilibrium-stress models

  - Assume constant-flux layer

  - Use log-law to model turbulent fluxes between first grid point and solid wall

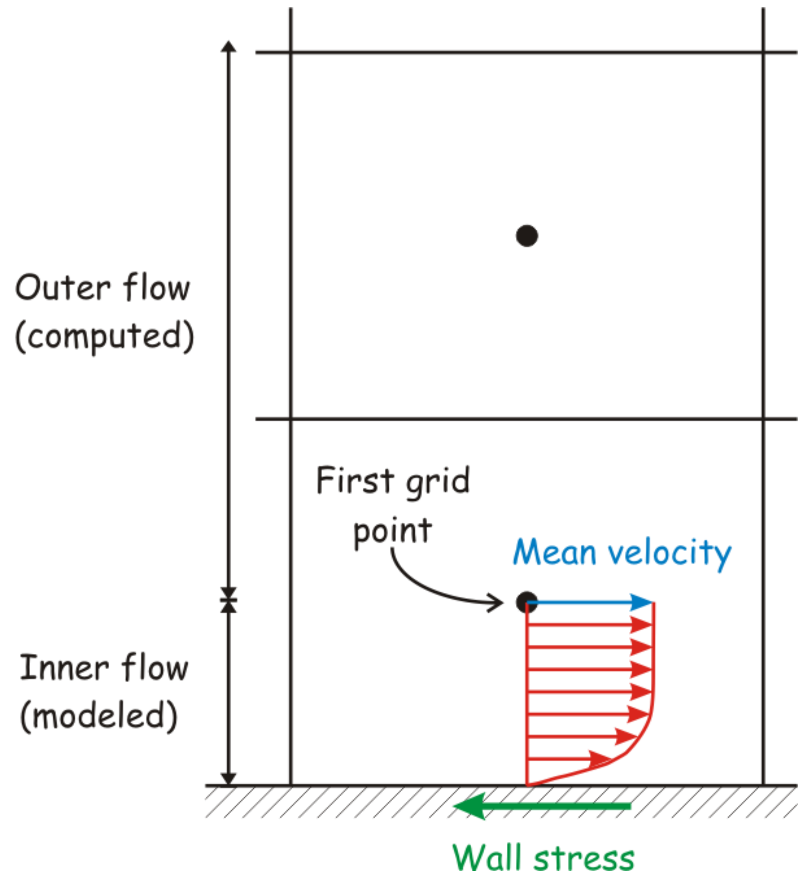


Figure adapted from Piomelli (PAS-1999)

$$\frac{U}{u_*} = \frac{1}{\kappa} \ln\left(\frac{z}{z_o}\right)$$

Log-law for a rough surface

Replace ν/u_* by roughness length scale z_o

Monin-Obukhov Similarity Theory

- In the atmospheric surface layer the log-law has to be modified to account for effects due to temperature stratification
- Monin and Obukhov (1954):

  "*In the surface layer (constant flux layer) the structure of the turbulence is determined by a few key parameters*:"

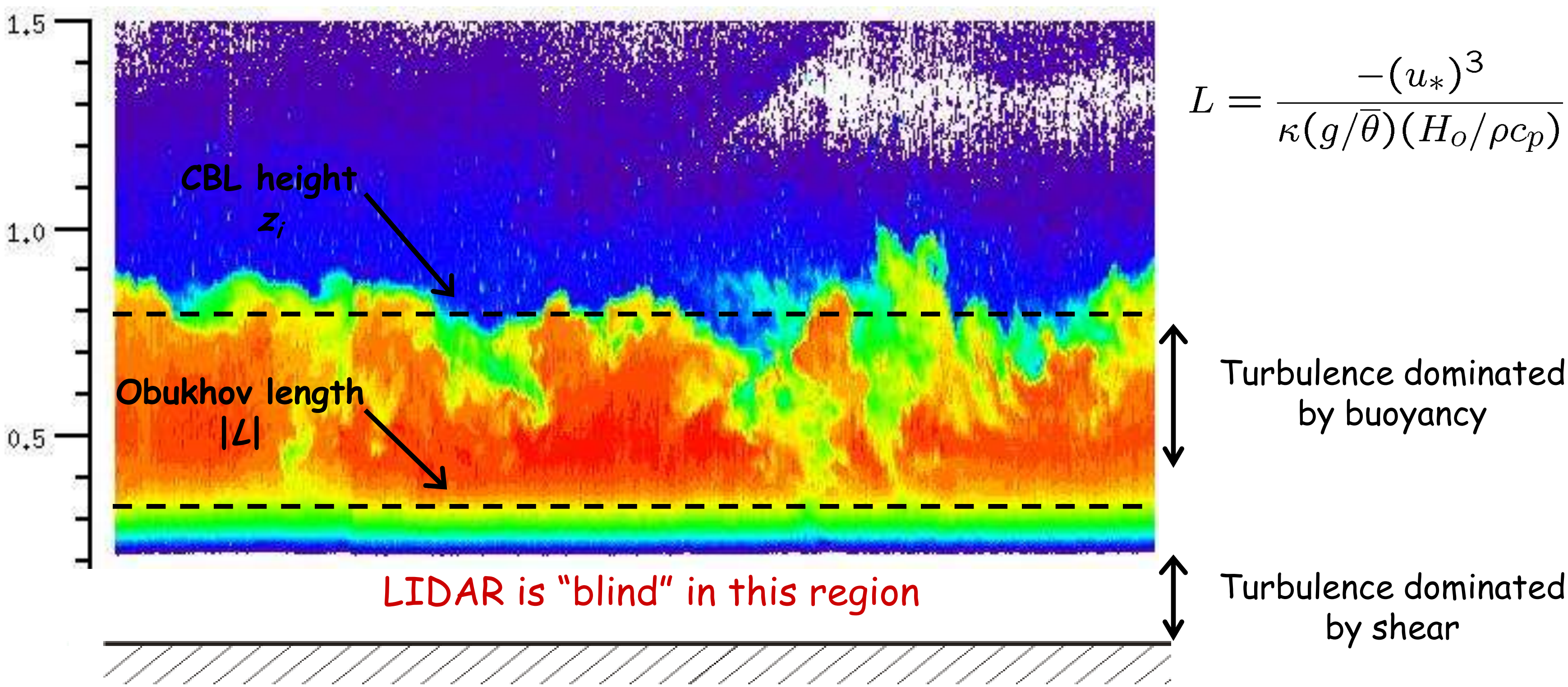
  - Height above the ground: z
  - Kinematic surface stress: $\tau_o/\rho = u_*^2$
  - Kinematic surface heat flux: $H_o/(\rho c_p)$
  - Stability parameter: $g/\overline{\theta}$
- Obukhov (1946): Obukhov length $L = \dfrac{-(u_*)^3}{\kappa(g/\overline{\theta})(H_o/\rho c_p)}$

Monin-Obukhov Similarity Theory (cnt'd)

- Lilly (1968): "$|L|$ *is an upper bound to the height at which turbulence is strongly influenced by surface shear*"

LIDAR image of the convective boundary layer (CBL) around noon

$$L = \frac{-(u_*)^3}{\kappa(g/\overline{\theta})(H_o/\rho c_p)}$$



Monin-Obukhov Similarity Theory (cnt'd)

- Dimensional analysis:

  - neutral temperature stratification:

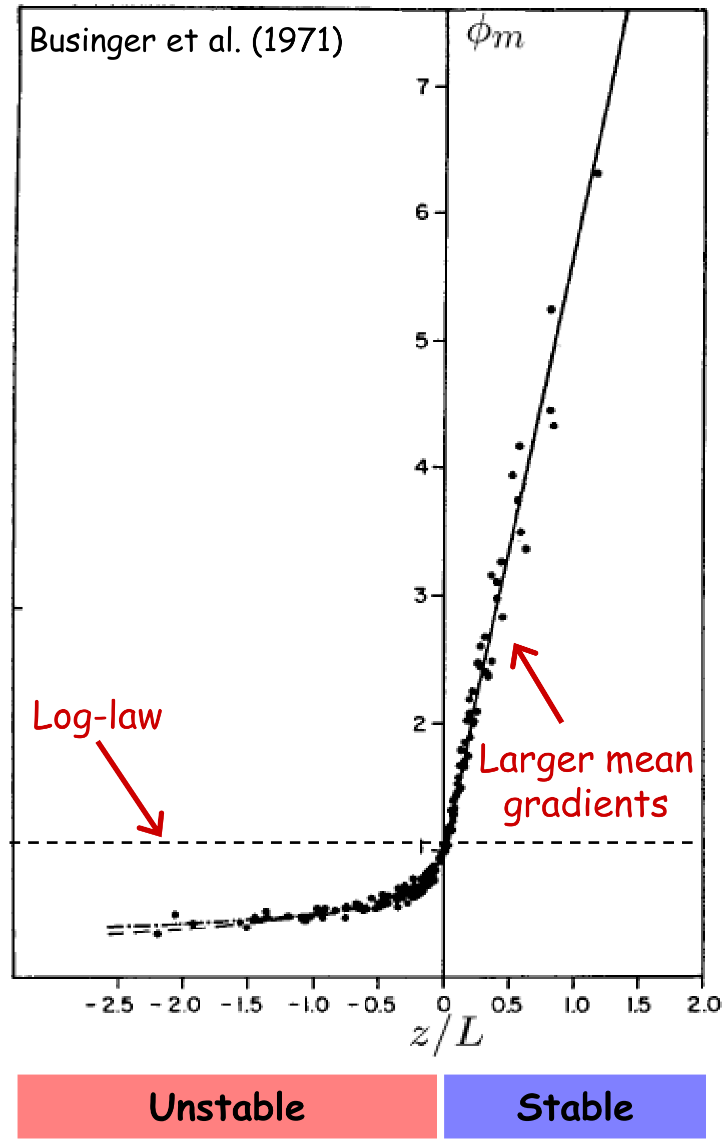
$$\frac{d\overline{u}}{dz} = f(z, u_*) \Rightarrow \frac{z}{u_*}\frac{d\overline{u}}{dz} = \frac{1}{\kappa} \Rightarrow \frac{\overline{u}}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_o}\right)$$

  - non-neutral temperature stratification:

$$\frac{d\overline{u}}{dz} = f(z, u_*, L) \Rightarrow \frac{\kappa z}{u_*}\frac{d\overline{u}}{dz} = \underbrace{\phi_m\left(\frac{z}{L}\right)}_{\text{M-O Similarity function}}$$

- From experimental data:

$$\phi_m\left(\frac{z}{L}\right) = \begin{cases} (1-16z/L)^{-1/4} & \text{if } z/L < 0 \\ 1+5z/L & \text{if } z/L > 0 \end{cases}$$



Monin-Obukhov Similarity Theory (cnt'd)

- Mean velocity profile:

- from MOS Theory:

$$\frac{\kappa z}{u_*}\frac{d\overline{u}}{dz} = \phi_m\left(\frac{z}{L}\right)$$

- Integrating:

$$\frac{\overline{u}}{u_*} = \frac{1}{\kappa}\left[\ln\left(\frac{z}{z_o}\right) - \Psi_m\left(\frac{z}{L}\right)\right]$$

Stability correction
(deviation from log-law)

$$\Psi_m\left(\frac{z}{L}\right) = \int \frac{1-\phi_m(z/L)}{z/L} d(z/L)$$

- Equilibrium stress model:

$$u_* = \left[\frac{\kappa \widetilde{u}_h}{\ln\left(\frac{z}{z_o}\right) - \Psi_m\left(\frac{z}{L}\right)}\right]$$

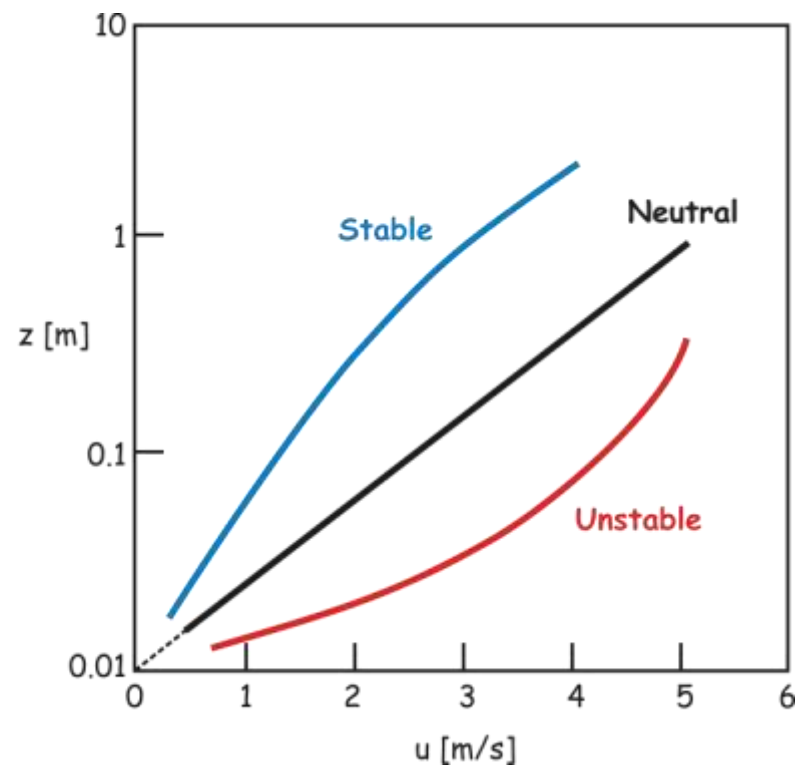


Figure adapted from Kaimal & Finnigan (1994)

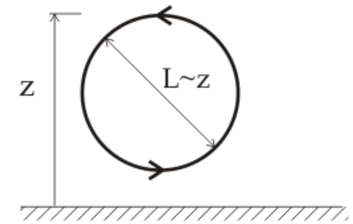
Subgrid Scale (SGS) Models for ABL flows

- Two main problems:

  - Complex physics introduced by mean shear and buoyancy effects

  - Poor resolution at the first vertical grid levels

    (filter in the production range)



- Some possible approaches:

  - Prescribe effects (e.g. Canuto and Cheng, 1997)

  - "Second-order closures" (e.g. Wyngaard, 2004)

  - Dynamic models based on Germano Identity

- As an example here: Smagorinsky and dynamic Smagorinsky models

$$\tau_{ij}^{smag} = -2(c_s\Delta)^2|\tilde{S}|\tilde{S}_{ij}$$

The dynamic Smagorinsky model

- SGS stress at grid-filter scale (Δ): $\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$
- SGS stress at test-filter scale ($\alpha\Delta$): $T_{ij} = \overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$
- Germano Identity: $L_{ij} \equiv \overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = T_{ij} - \bar{\tau}_{ij}$
- Smagorinsky model at both scales:

$$\tau_{ij}^{smag} = -2[c_s(\Delta)\Delta]^2 |\tilde{\mathrm{S}}| \tilde{S}_{ij} \qquad T_{ij}^{smag} = -2[c_s(\alpha\Delta)\alpha\Delta]^2 |\bar{\tilde{\mathrm{S}}}| \bar{\tilde{S}}_{ij}$$

- Define the error of the model as: $\epsilon \equiv L_{ij} - (T_{ij}^{smag} - \bar{\tau}_{ij}^{smag})$
- Assume scale similarity: $c_s(\alpha\Delta) = c_s(\Delta)$

← Can be justified for filters in the inertial range

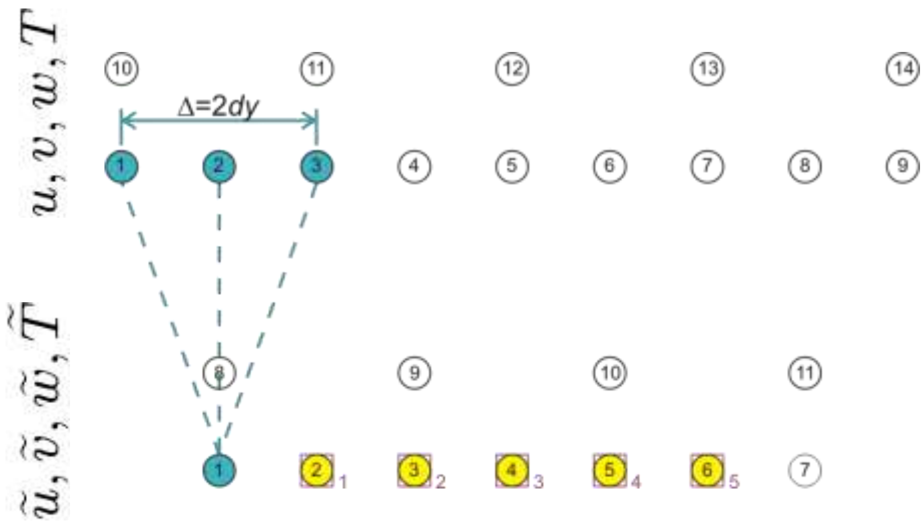
- Minimizing least squared error yields:

$$(c_s^{\Delta})^2 = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{mn} M_{mn} \rangle} \qquad M_{ij} = 2\Delta^2 (\overline{|\tilde{\mathrm{S}}| \tilde{S}_{ij}} - \alpha^2 |\bar{\tilde{\mathrm{S}}}| \bar{\tilde{S}}_{ij})$$

A priori SGS model tests for ABL flows

Kettleman City, CA, 2000
HATS campaign (NCAR-JHU)

- 2D filter (using Taylor's hypothesis):



- Determine c_s^2 by requiring the model to predict the correct dissipation:

$$c_s^2 = \frac{\langle -\tau_{ij}\tilde{S}_{ij}\rangle}{\langle 2\Delta^2|\tilde{S}|\tilde{S}_{ij}\tilde{S}_{ij}\rangle}$$

Measured energy transfer to SGS scales

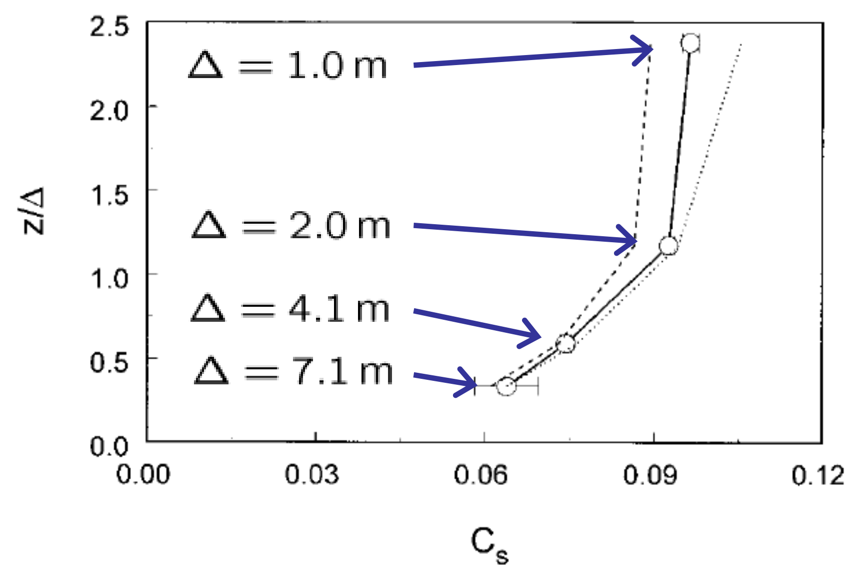
Smagorinsky energy dissipation

Experimental evidence for scale dependence

- Scale similarity: Cs can depend on height (z) but not on filter width (Δ)
- Measurements at a constant height above ground (z_1=3.41m and z_2=3.92m)
- *A priori* analysis using different filter widths

UC Davis, CA, Summer 1999
Porte-Agel et al. (2001)

Cs should be constant for scale-similarity to hold



Cs is scale-dependent!

A Scale-dependent dynamic model

- Dynamic model:

  - Assume $c_s(\alpha\Delta) = c_s(\Delta) = c_s$

  - Use Germano identity to determine c_s

- Scale-dependent dynamic model:

  (Porte-Agel et al., 2000; Bou-Zeid et al., 2005)

  - Assumption: power-law dependence $c_s(\Delta) = m\Delta^{\phi}$

  - Approach:

    a) Apply 2 test-filters, typically at scales $\alpha\Delta$ and $\alpha^2\Delta$

    b) Use Germano identity twice to solve for m and ϕ

Experimental validation

- Power-law dependence implies:

$$\frac{c_s(\alpha^2\Delta)}{c_s(\alpha\Delta)} = \frac{m(\alpha^2\Delta)^\phi}{m(\alpha\Delta)^\phi} = \alpha^\phi$$

$$\frac{c_s(\alpha\Delta)}{c_s(\Delta)} = \frac{m(\alpha\Delta)^\phi}{m(\Delta)^\phi} = \alpha^\phi$$

- Test of validity:

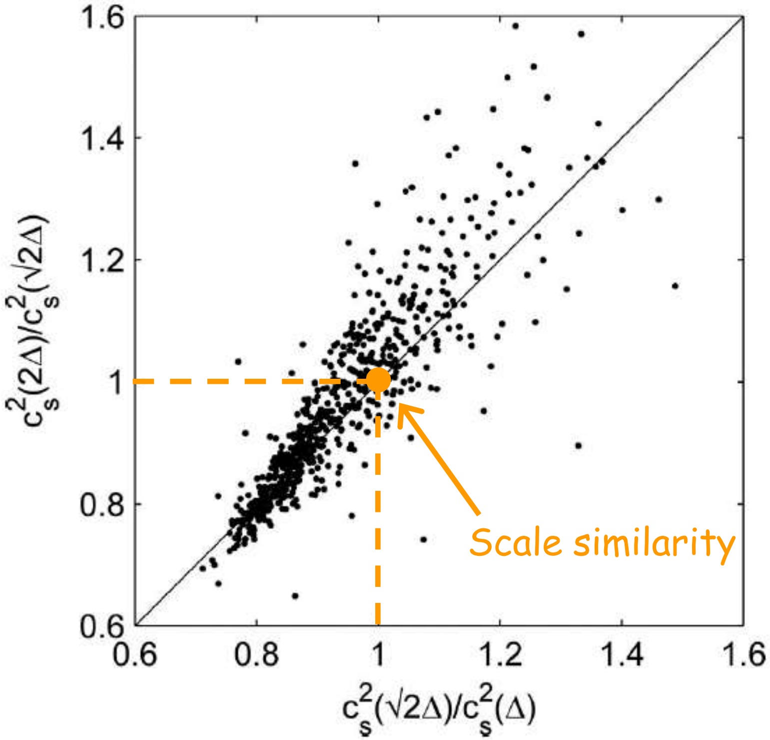
$$\frac{c_s(\alpha^2\Delta)}{c_s(\alpha\Delta)} = \frac{c_s(\alpha\Delta)}{c_s(\Delta)}$$

- For scale similarity to hold:

$$c_s(\Delta) = m\Delta^\phi = c_s^* \Rightarrow \phi = 0$$

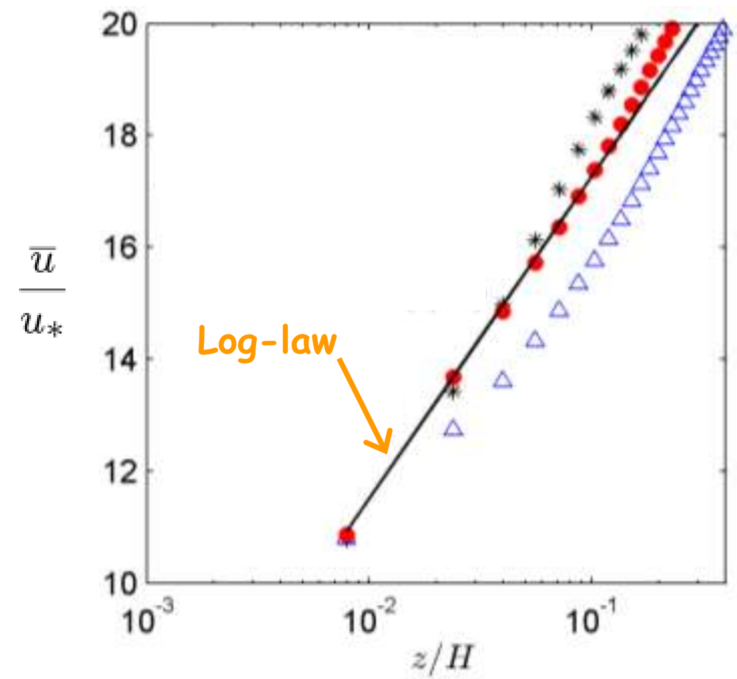
$$\phi = 0 \Rightarrow \frac{c_s(\alpha^2\Delta)}{c_s(\alpha\Delta)} = \frac{c_s(\alpha\Delta)}{c_s(\Delta)} = 1$$

Lake Geneva, Switz., 2006
Bou-Zeid et al. (2008)



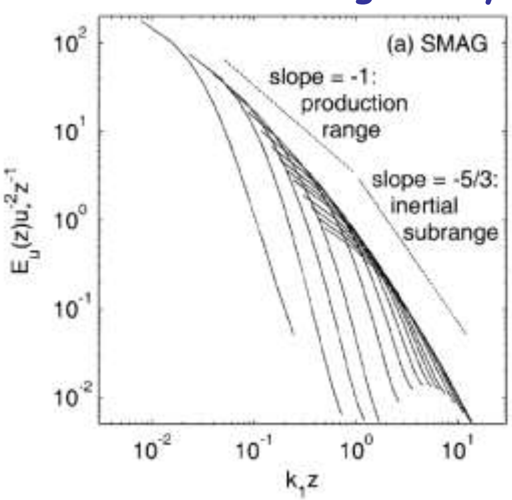
A posteriori validation for high-Re channel flow

- Comparison of SGS models:
  - * Standard Smagorinsky
  - △ Dynamic model (scale similar)
  - ● Scale-dependent dynamic model

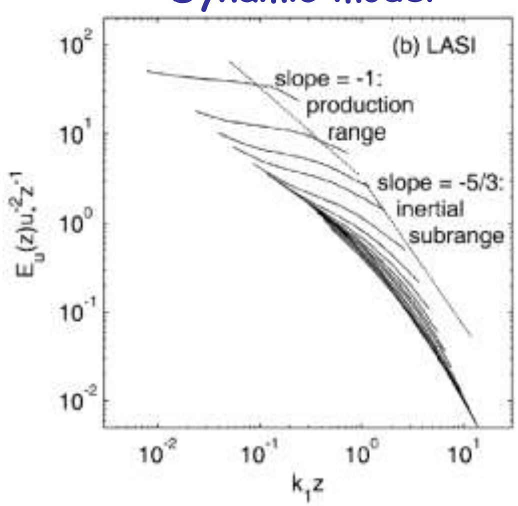


- Spectrum:

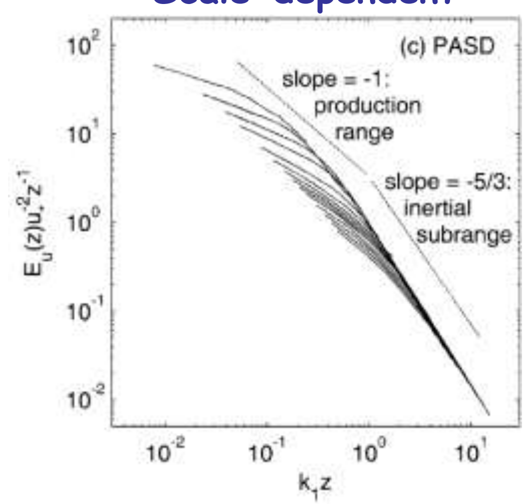
Standard Smagorinsky



Dynamic model



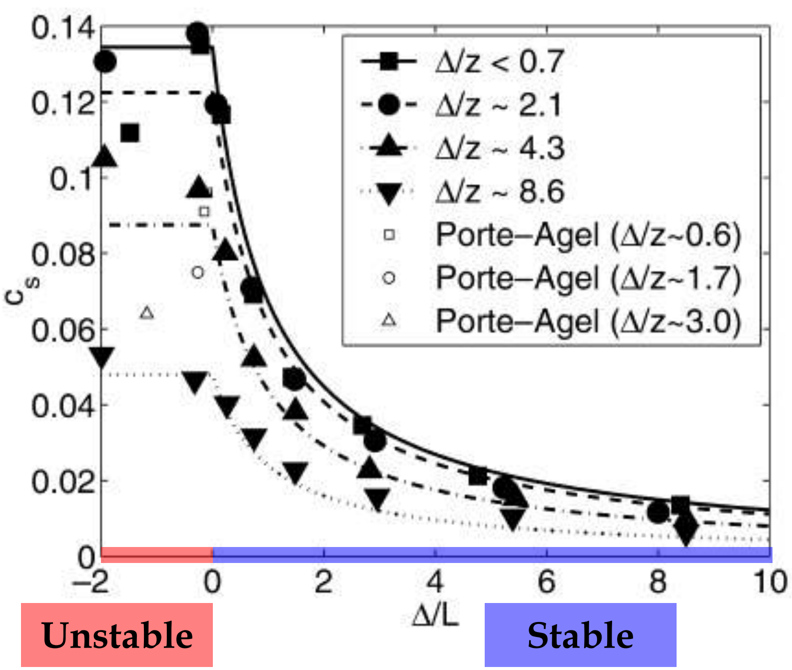
Scale-dependent



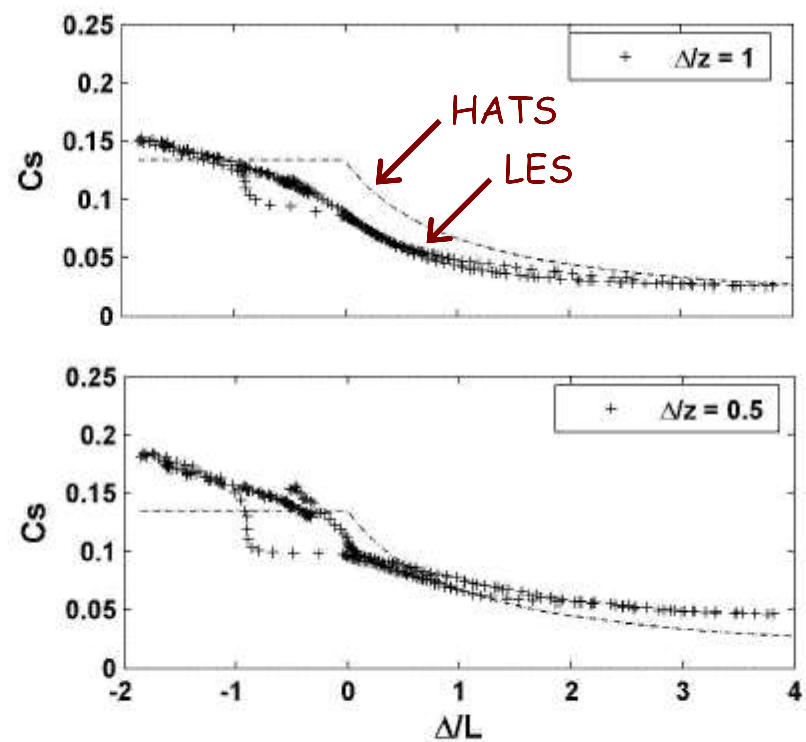
Buoyancy effects on model coefficient

- Buoyancy has a profound effect on interactions among scales in ABL flows
- Model constants will depend on buoyancy/shear balances in models that do not account for these effects explicitly (Chamecki et al., 2007)

A priori analysis - Kleissl et al (2003)

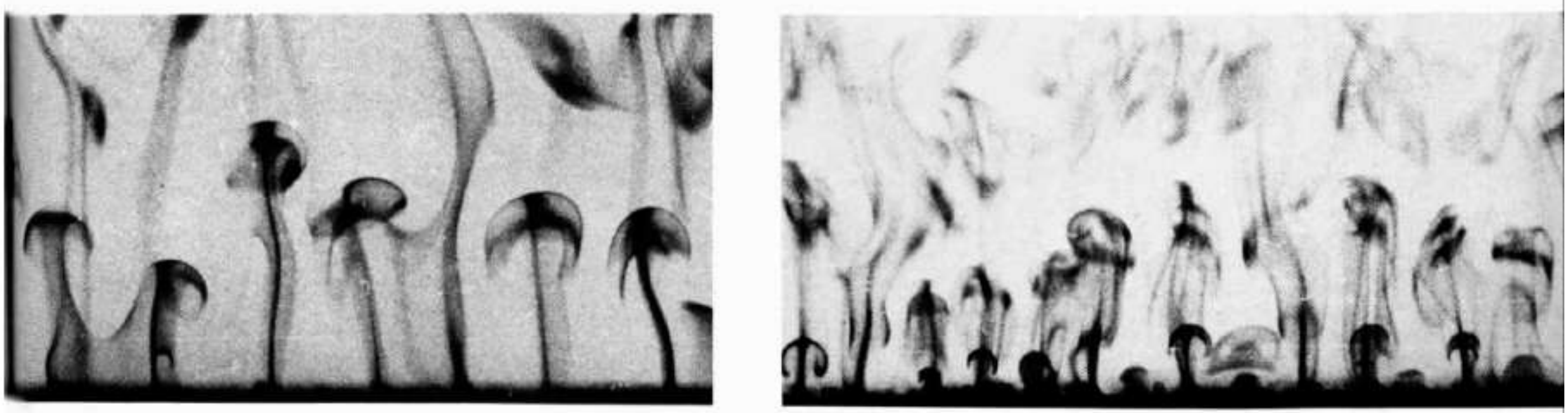


A posteriori results - Kumar et al (2006)



Applications – The convective boundary layer

- Strong surface heating and weak mean wind
- Buoyancy dominates the dynamics, shear is not important (L is very small)
- Flow is dominated by large coherent structures (thermal plumes)



108. Buoyant thermals rising from a heated surface. Mushroom-shaped plumes rise periodically above a heated copper plate. They are made visible by an electrochemical technique using thymol blue. The heating rate is higher in the photograph at the right. *Sparrow, Husar & Goldstein 1970*

An album of fluid motion (Van Dyke)

- Important consequences for atmospheric dispersion

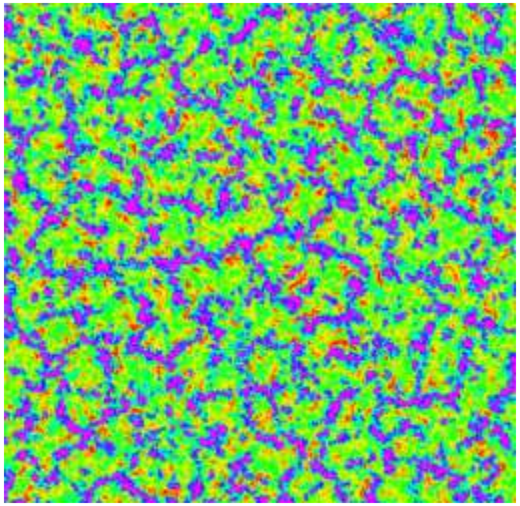
  - Increased vertical mixing

  - Asymmetric distribution of vertical velocity

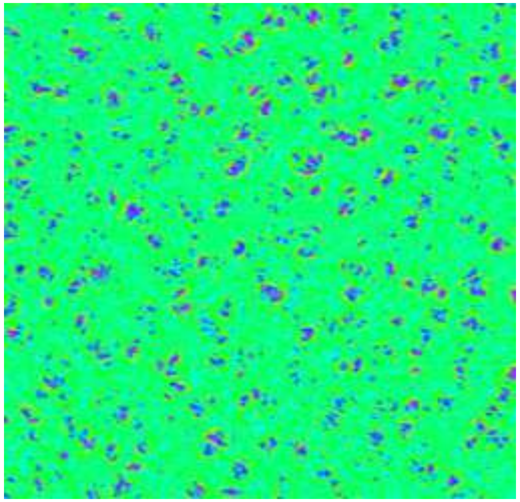
LES of convective boundary layer

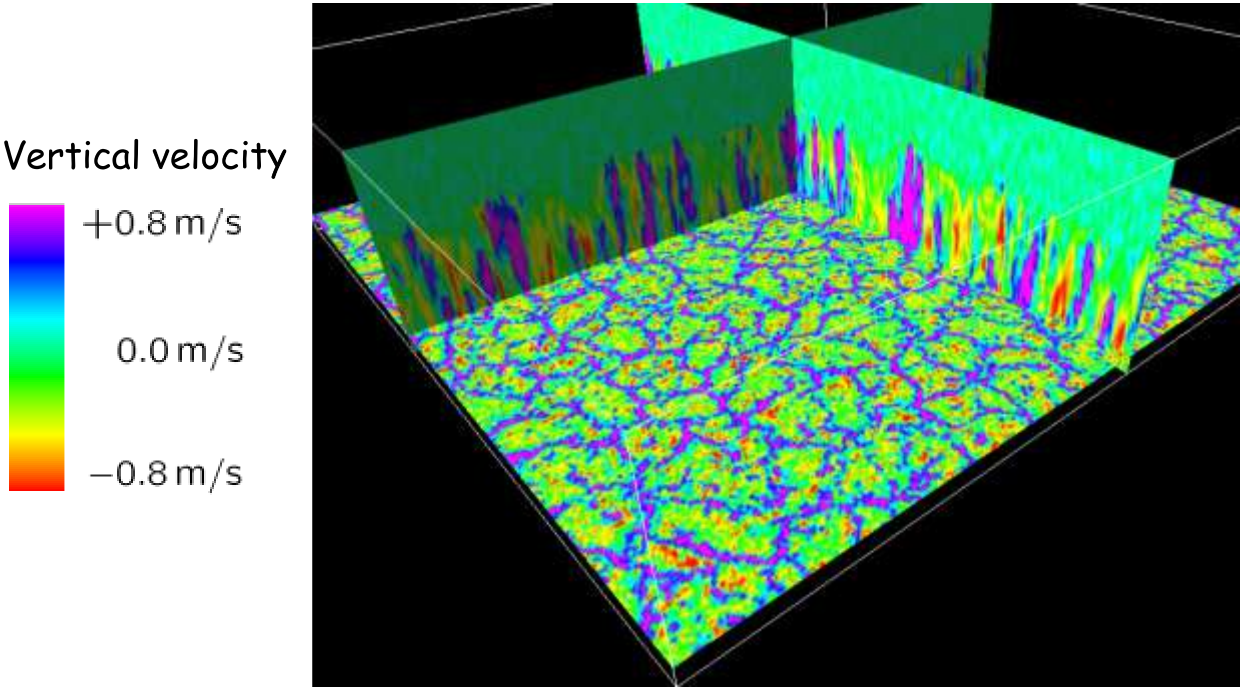
- Simulation from Peter Sullivan (NCAR)
- Strong surface heating and no mean wind
- Domain: 20 x 20 x 1.5 km^3 (400 x 400 x 96 grid points)

Horizontal cut at z=H/4



Horizontal cut at z=H/2





Dispersion in convective boundary layers

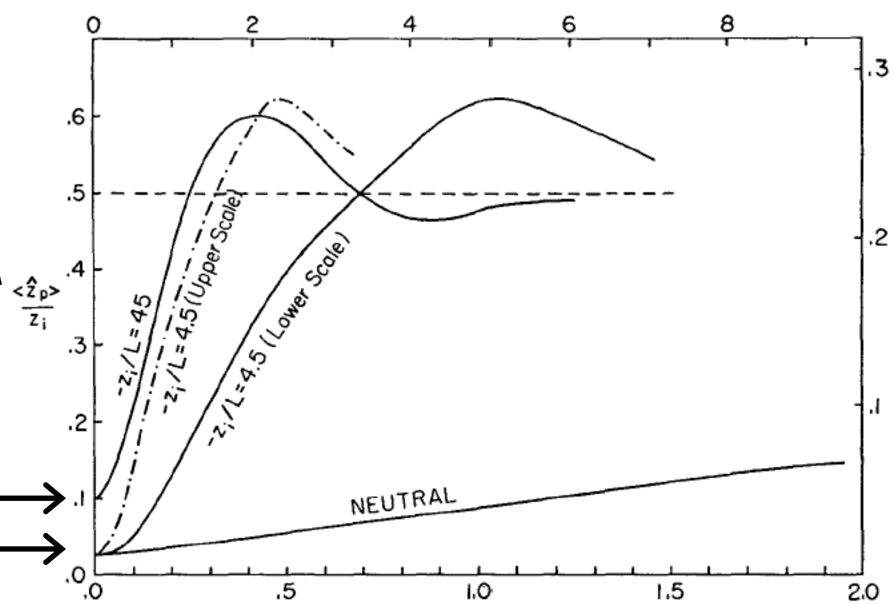
JANUARY 1972 JAMES W. DEARDORFF

Numerical Investigation of Neutral and Unstable Planetary Boundary Layers

JAMES W. DEARDORFF
National Center for Atmospheric Research,[1] Boulder, Colo. 80302

- Major contribution to understanding of the CBL (scaling, eddy structure, etc.)
- 4 paragraphs on dispersion in the CBL at the very end

Mean height of particles released near the surface at t=0



Source height

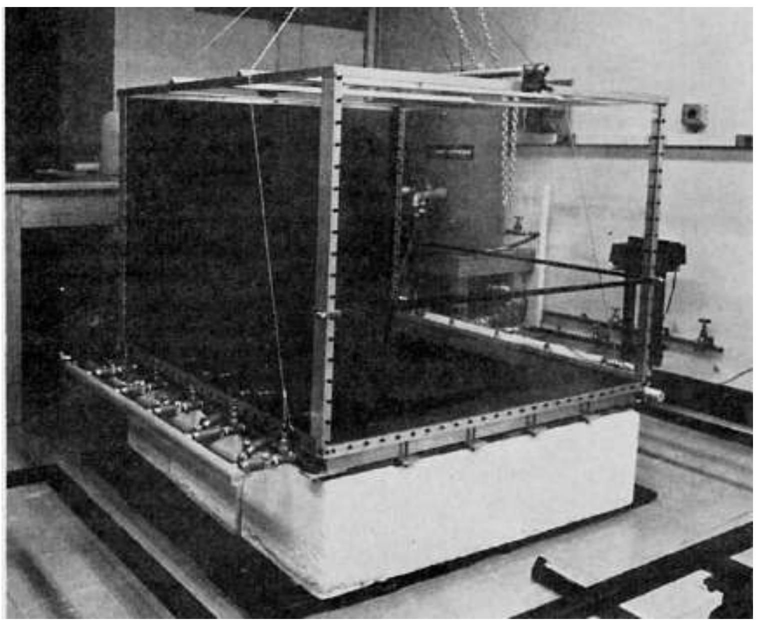
Dimensionless time

Main conclusions:

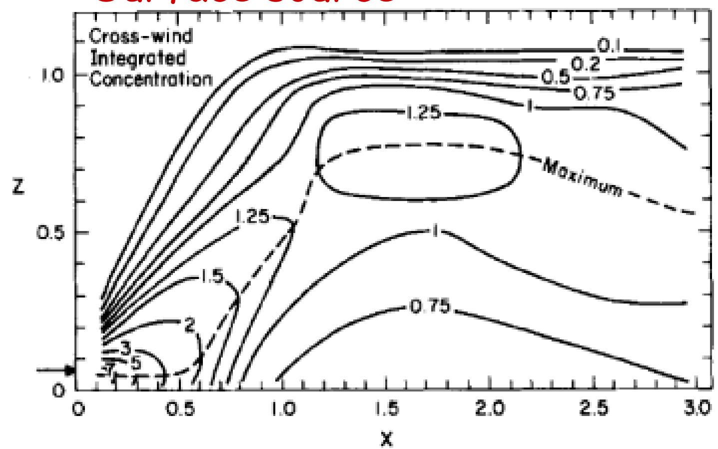
1) Cluster rises fast (on updrafts)
2) Cluster overshoots equilibrium
3) Fast vertical spread (1 o2 two orders of magnitude faster than neutral)

Dispersion in convective boundary layers (cnt'd)

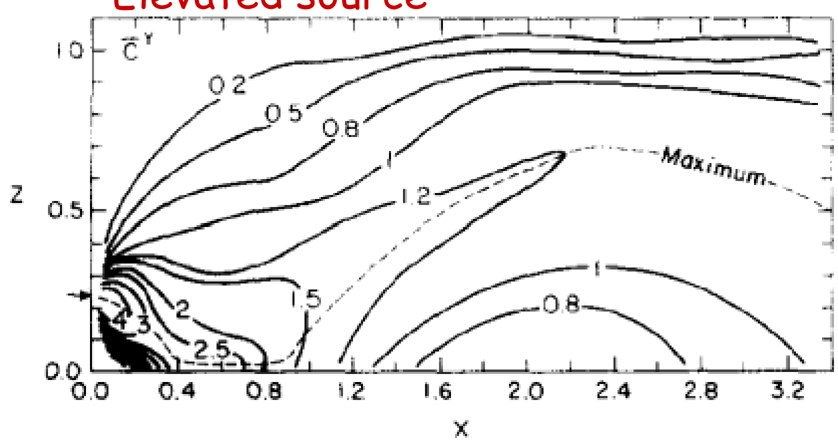
- Deardorff's LES results lead to a series of studies of dispersion in the CBL
- Laboratory experiments of point source release in convection chambers by Willis and Deardorff



Surface source



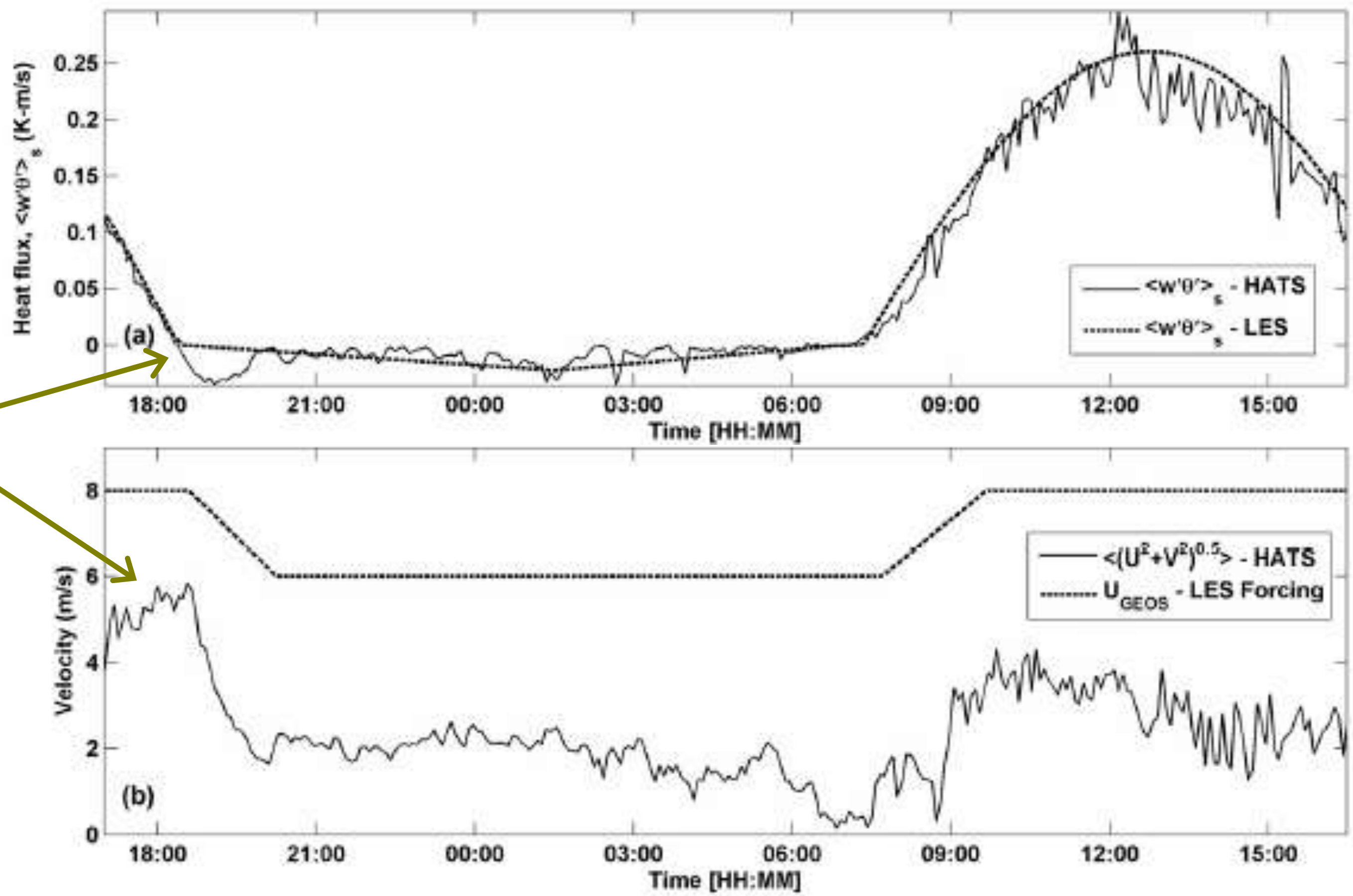
Elevated source



- Confirmed by LES (Lamb, 1978)
- CONDORS field experiment (1983)
- Adopted by EPA in 2005 (AERMOD)

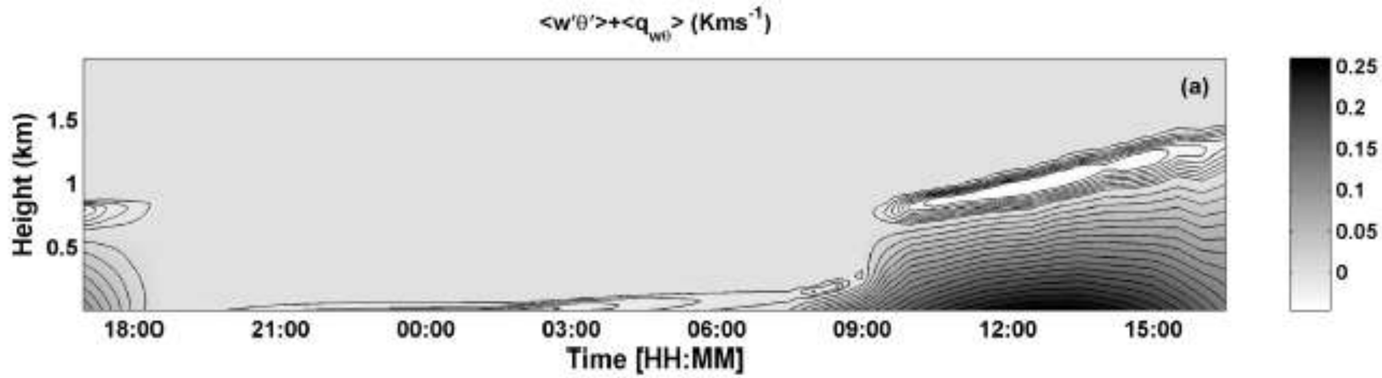
LES of the complete diurnal cycle

- Domain: 4 x 4 x 2 km^3 (160 x 160 x 160 grid points)
- Scale-dependent dynamic Smagorinky SGS model
- Forced using geostrophic velocity and surface heat flux from experimental data

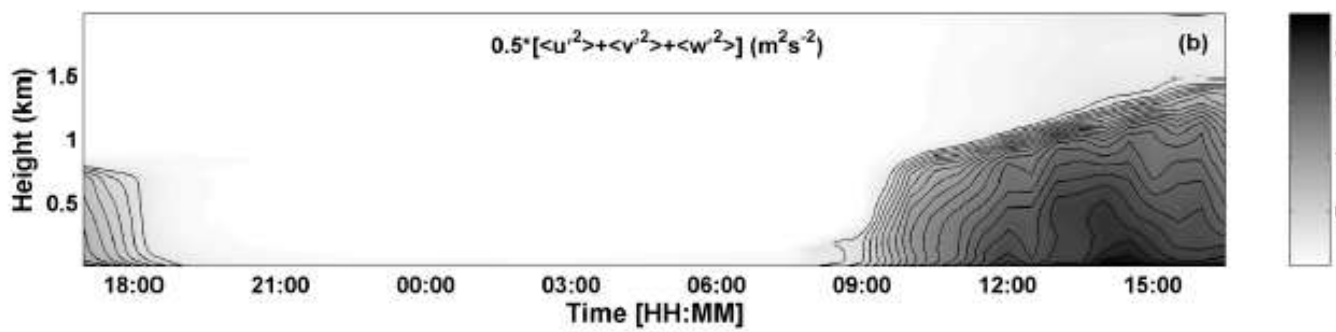


LES of the complete diurnal cycle (cnt'd)

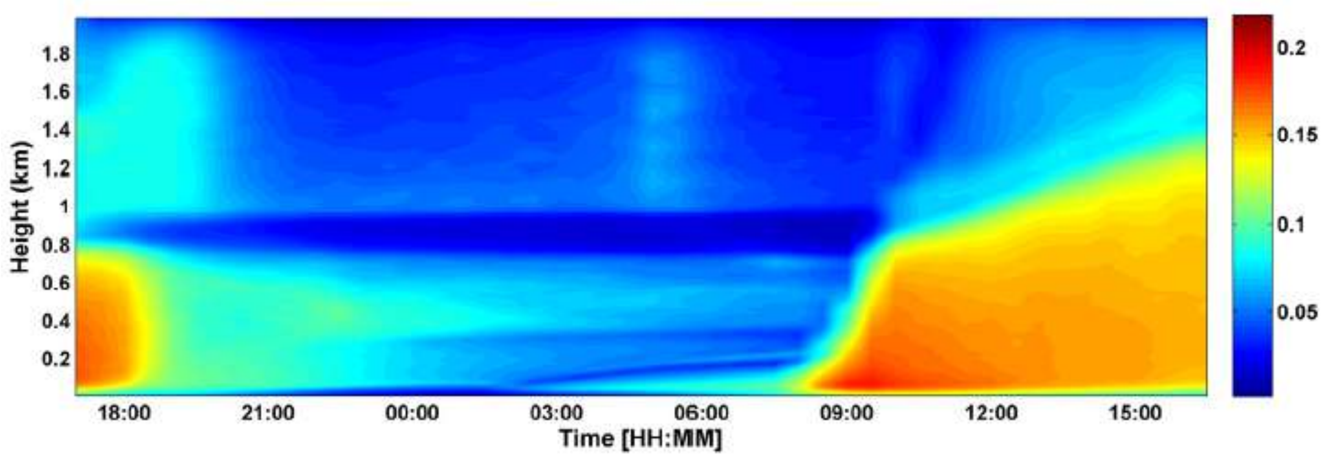
Total heat flux (resolved + SGS)



Resolved TKE

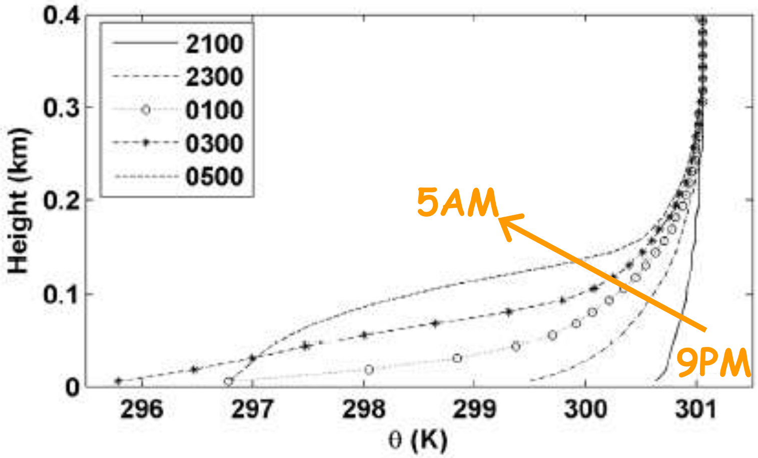


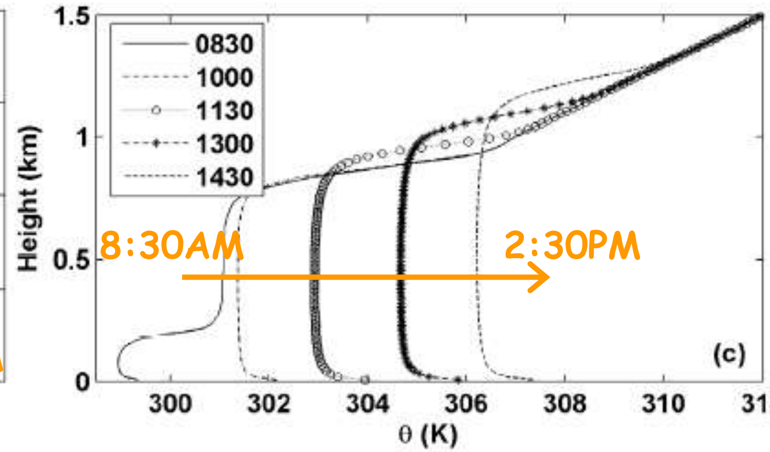
Dynamic Smagorisnky coefficient Cs



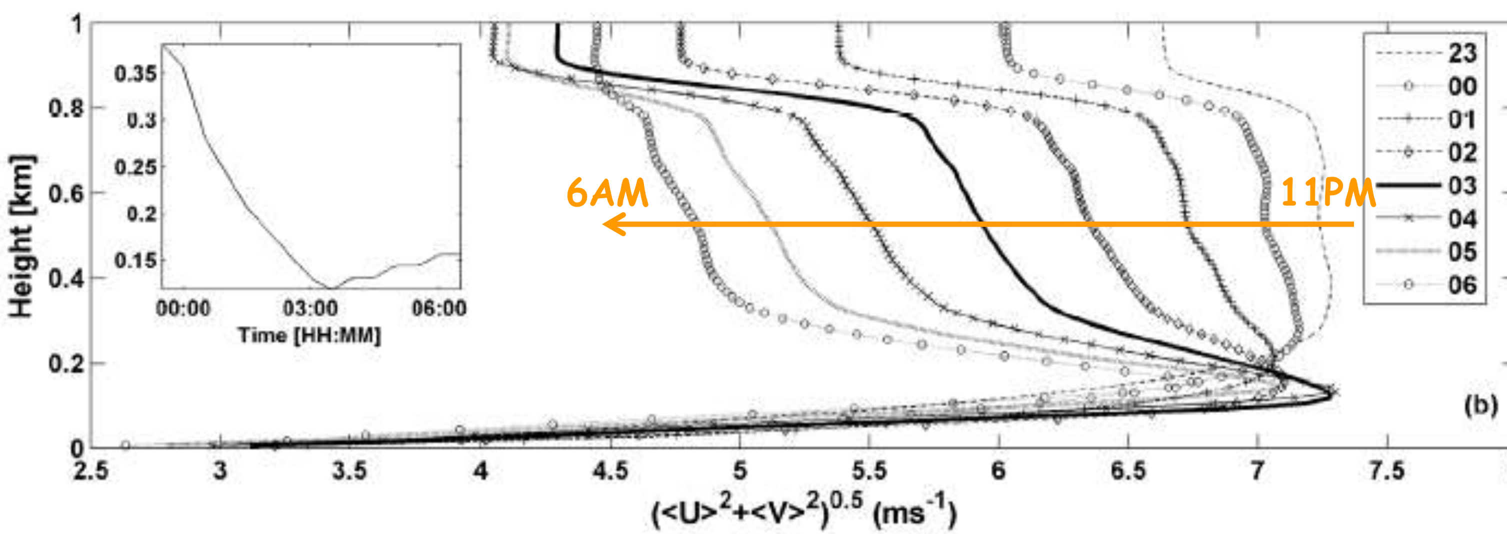
LES of the complete diurnal cycle (cnt'd)

- Evolution of stable and unstable ABL





- Evolution of the nocturnal low-level jet



Challenges: moving to more realistic conditions

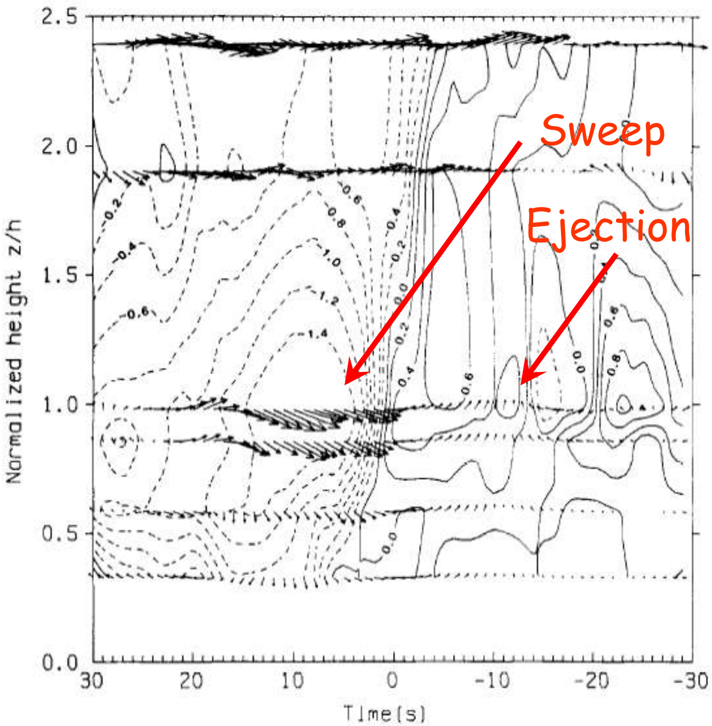
- These idealized studies are very useful in building up basic understanding of the dynamics of ABL flows
- Moving toward more "realistic" conditions require:

  - Replacing periodic horizontal b.c.'s by inflow from regional/mesoscale models
  - Coupling radiation and dynamics (surface energy balance, clouds, etc.)
  - Resolving vegetated surfaces (instead of modeling trees as roughness)
  - Accounting for topography
  - Resolving urban areas (buildings, etc.)
  - Including surface heterogeneity (resolved and subgrid)
  - …

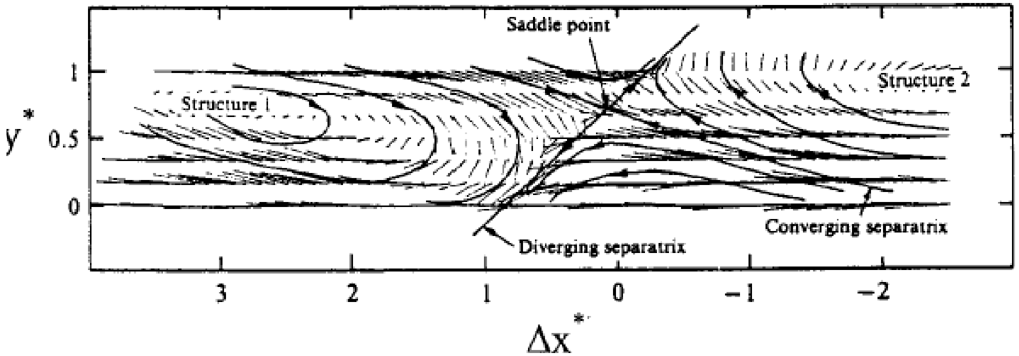
Turbulence above and within plant canopies

- Observations suggest existence of coherent structures:
  - Strong organized motions of short duration
  - Large contribution to fluxes of momentum/heat

- Gao et al. (1989) – measurements over forest:

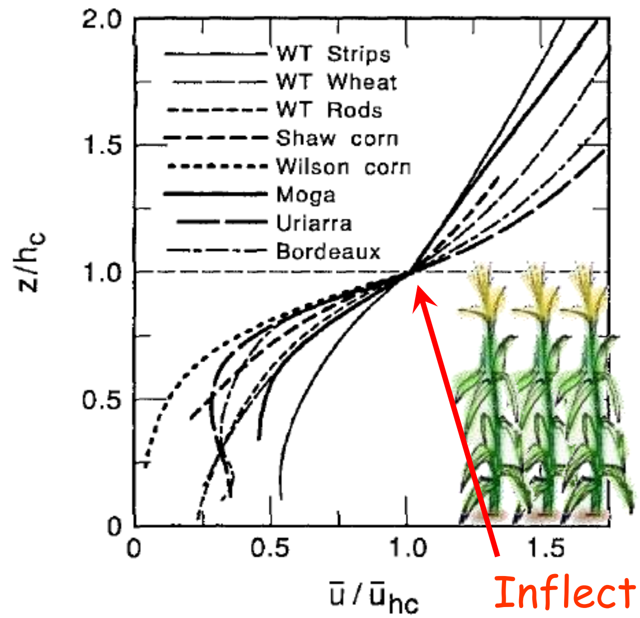


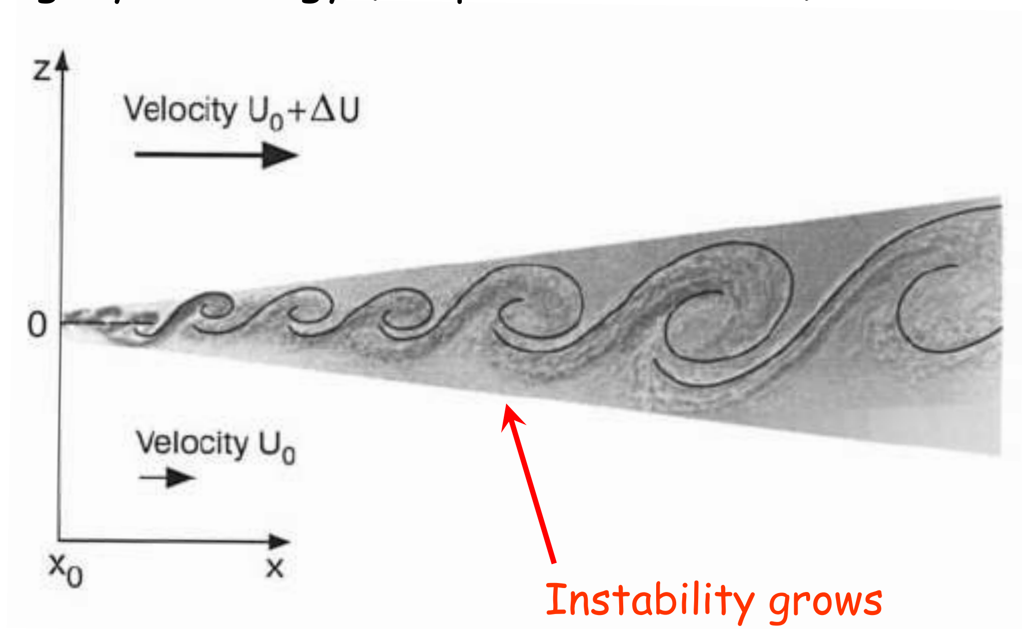
- Antonia et al. (1986) - ramp-cliff structures form at the diverging separatrix between two large eddies



Turbulence above and within plant canopies (cnt'd)

- Mean velocity profile and mixing layer analogy (Raupach et al., 1996):





Inflection point favors KH instability

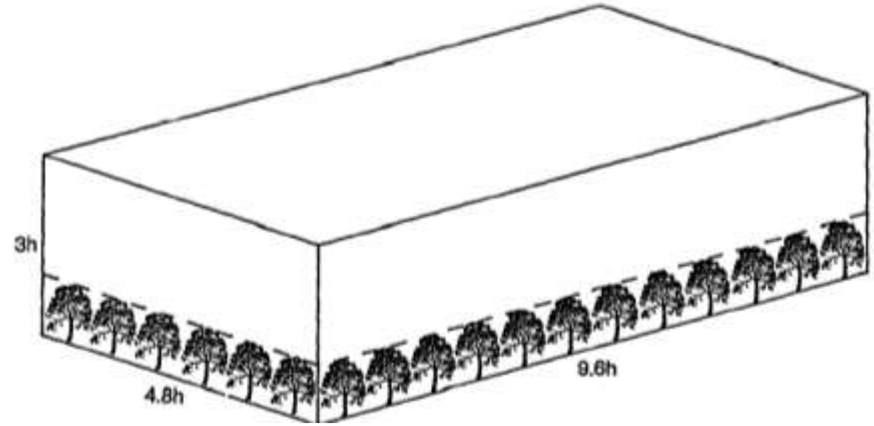
Instability grows quickly forming "coherent structures"

- Visual hint: waving cereal crops (mostly wheat) – "honami"
- Differences from ML: growth is limited by canopy and structures travel faster
- Experimental data lack spatial information required to fully understand the development and dynamic of these coherent structures

LES over vegetated surfaces

- Shaw and Schumann (1992):

  - Include drag by the canopy and heat input from vegetation



$$\nabla \cdot \tilde{\mathrm{u}} = 0$$

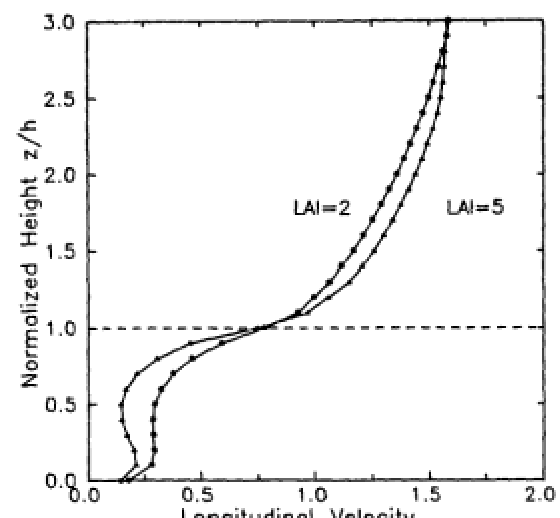
$$\frac{\partial \tilde{\mathrm{u}}}{\partial t} + \tilde{\mathrm{u}} \cdot \nabla \tilde{\mathrm{u}} = -\frac{1}{\rho} \nabla \tilde{p} + \left(1 - \frac{\tilde{\theta}}{\langle \tilde{\theta} \rangle}\right) \mathrm{g} - 2\Omega \times \tilde{\mathrm{u}} + \nu \nabla^2 \tilde{\mathrm{u}} - \nabla \cdot \tau - C_d A_f(z) |\tilde{\mathrm{u}}| \tilde{\mathrm{u}}$$

Drag coefficient

Frontal area density of vegetation

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{\mathrm{u}} \cdot \nabla \tilde{\theta} = \frac{\nu}{Pr} \nabla^2 \tilde{\theta} - \nabla \cdot \pi + S(z)$$

Heat transfer from vegetation

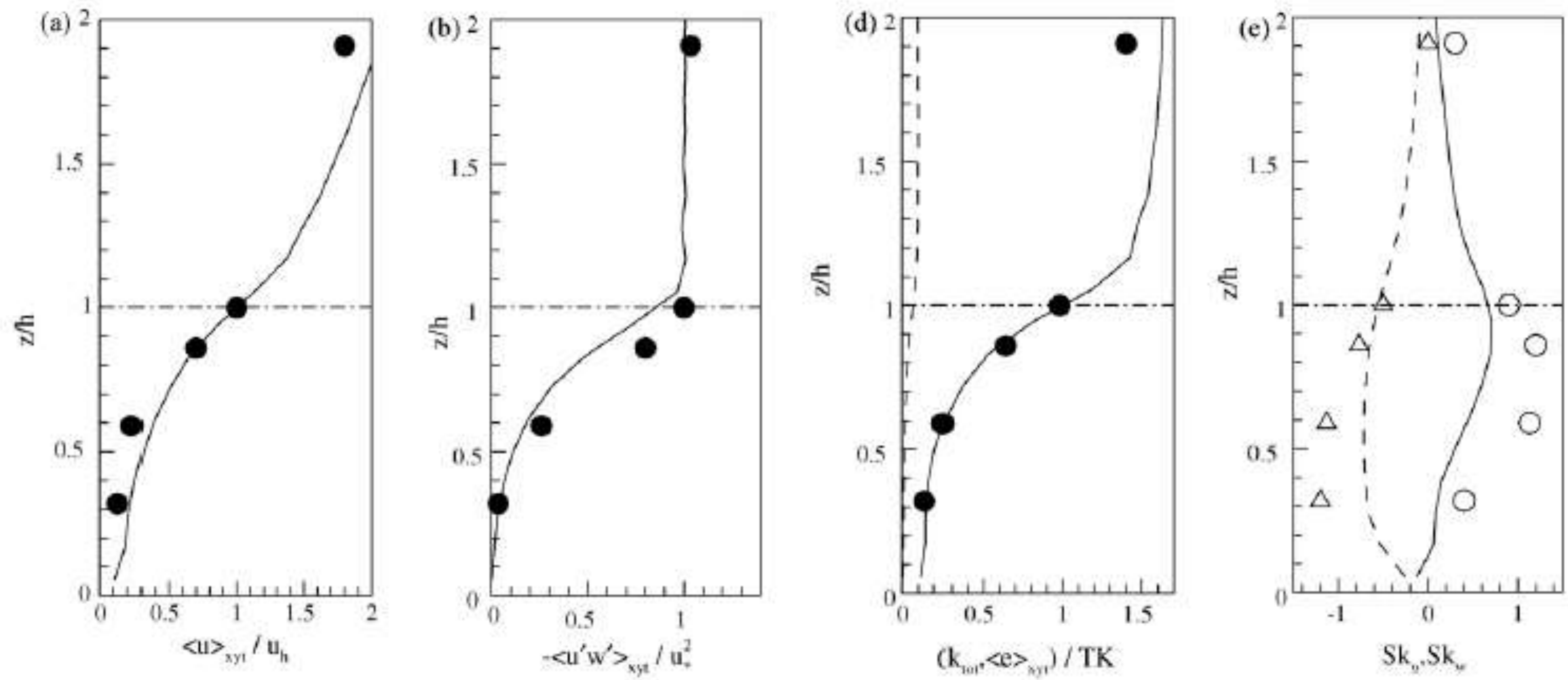


  - "Qualitative agreement with experiments"
  - Powerful tool to explore coherent structures (sweeps and ejections)

LES over vegetated surfaces (cnt'd)

- Detailed validation by Su et al. (1998) and Dupont and Brunet (2008)

Mean streamwise velocity | Momentum flux | Total TKE | Velocity Skewness



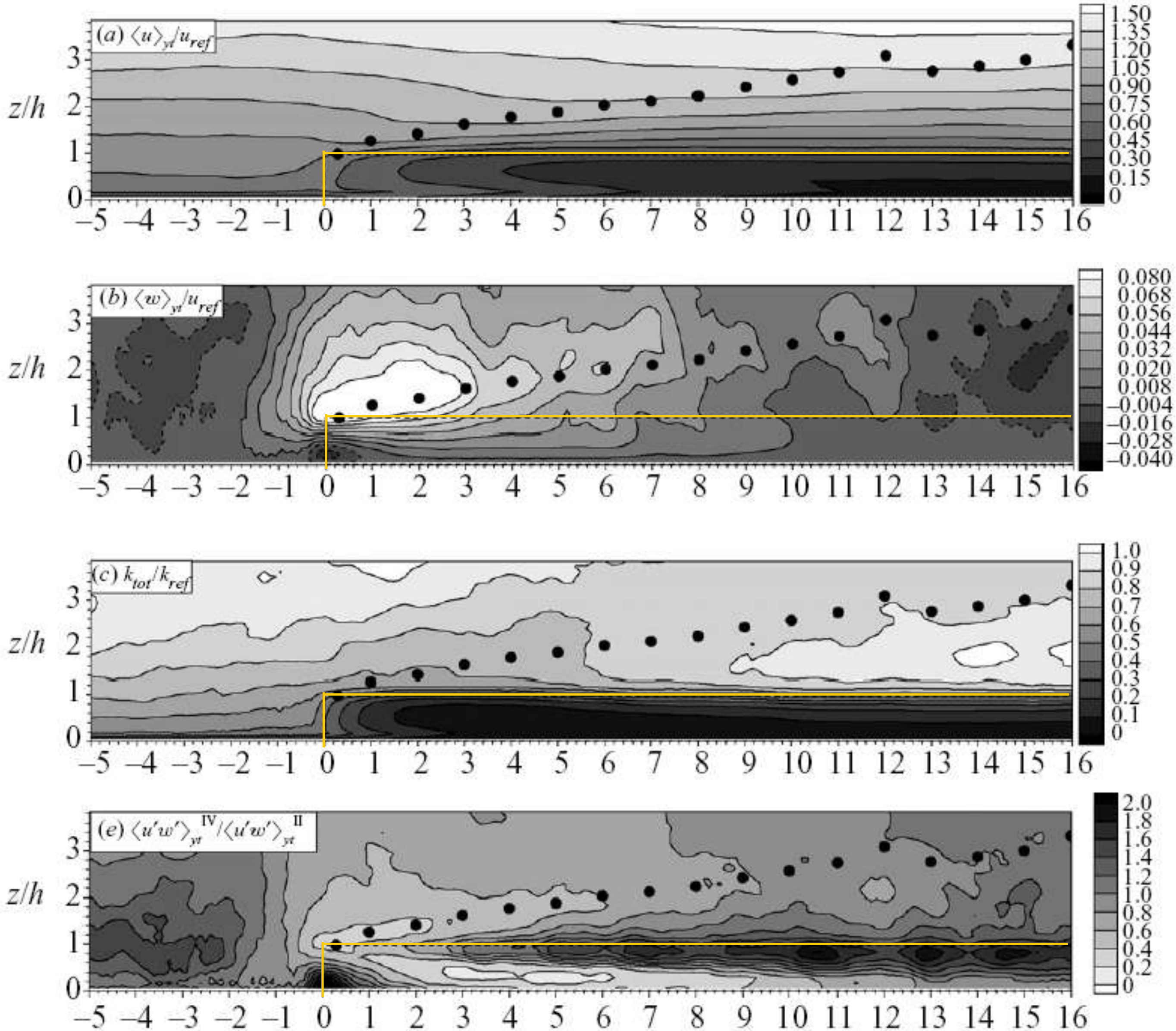
LES over vegetated surfaces (cnt'd)

Mean streamwise velocity

Mean vertical velocity

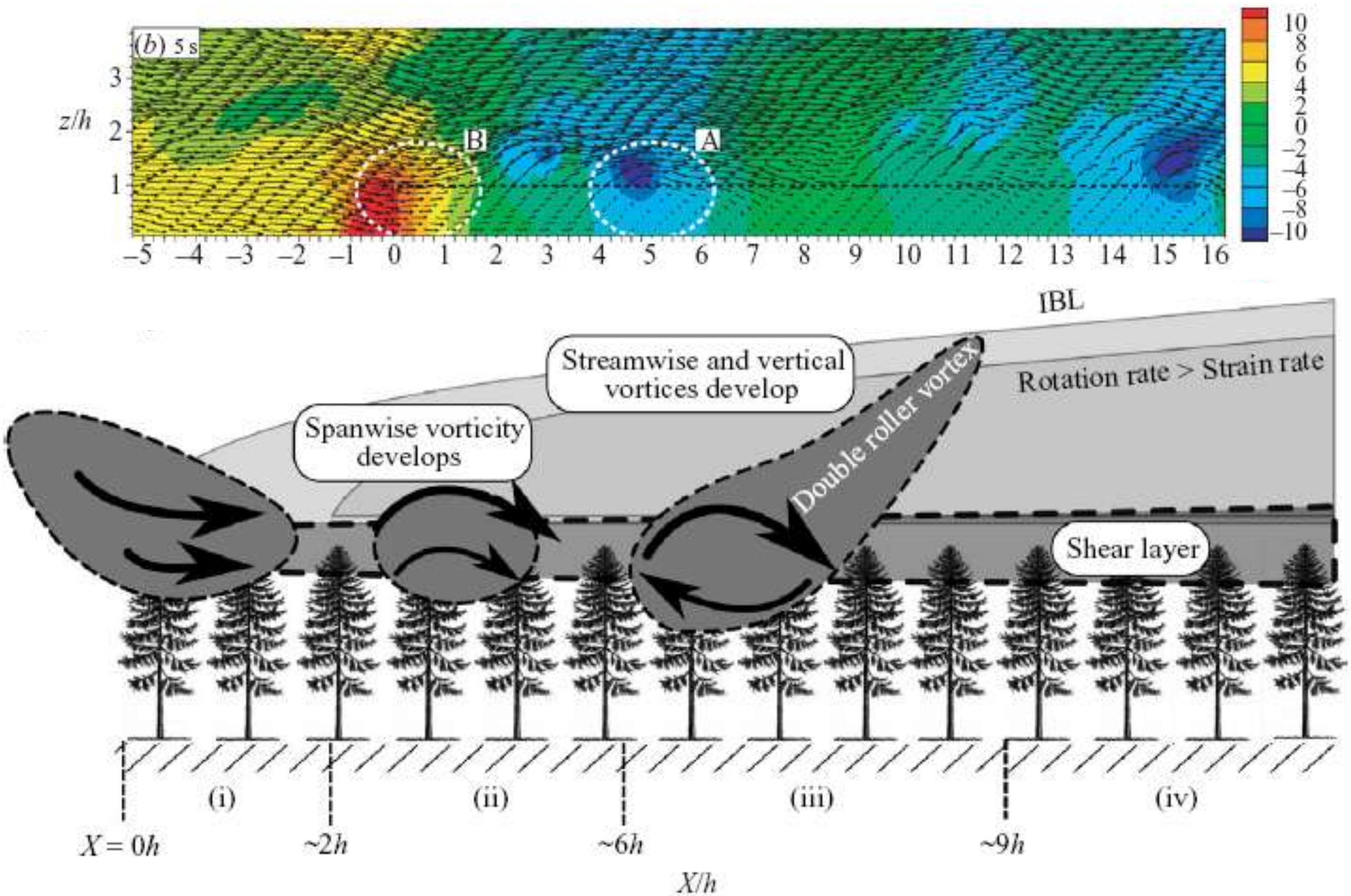
Total TKE

Sweeps/ejections



From Dupont and Brunet (2009)

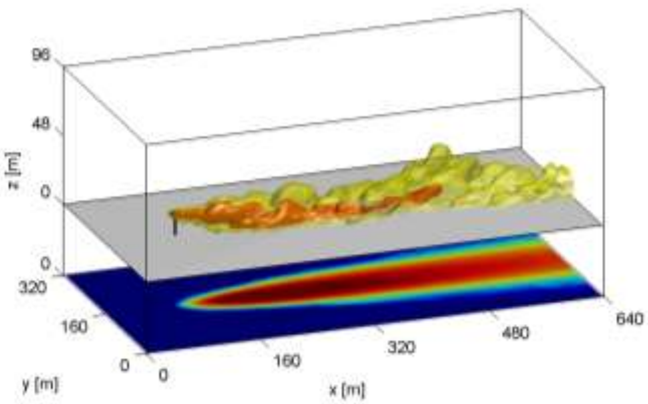
LES over vegetated surfaces (cnt'd)

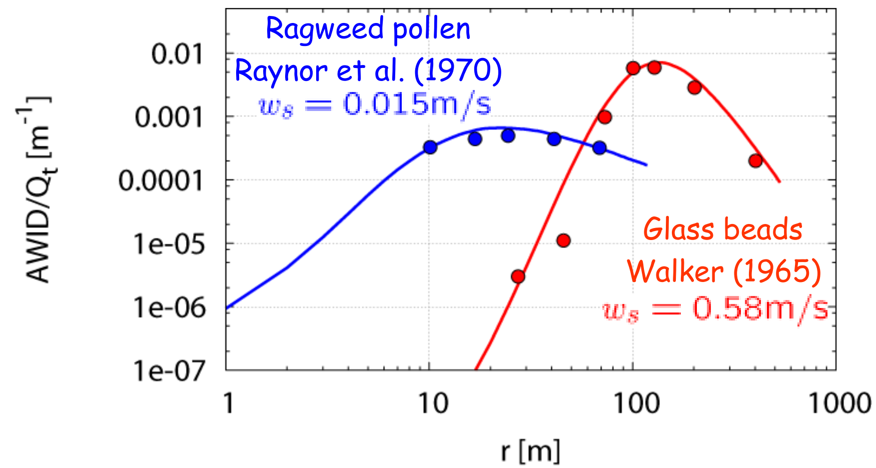


From **Dupont** and Brunet (2009)

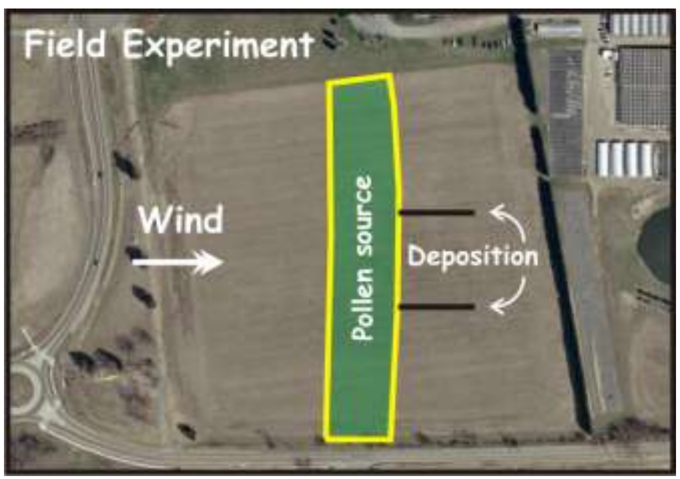
LES studies of pollen dispersion

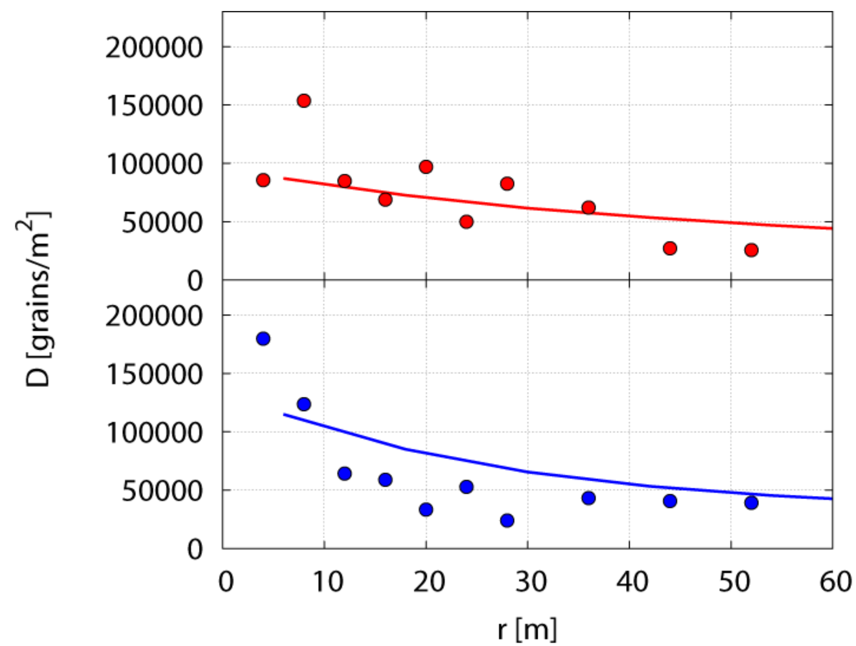
- Point source releases:





- Area source release of ragweed pollen:

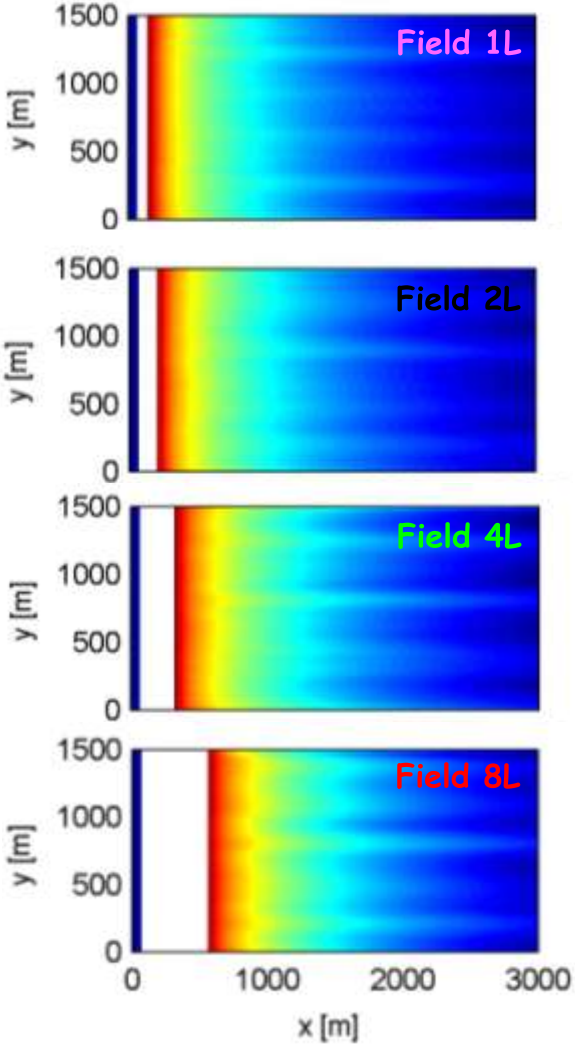


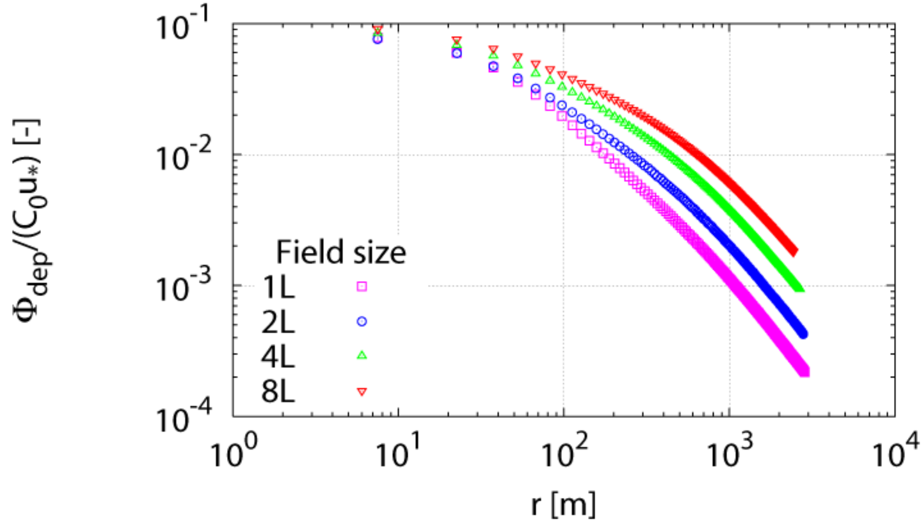


Chamecki et al. (2008, 2009)

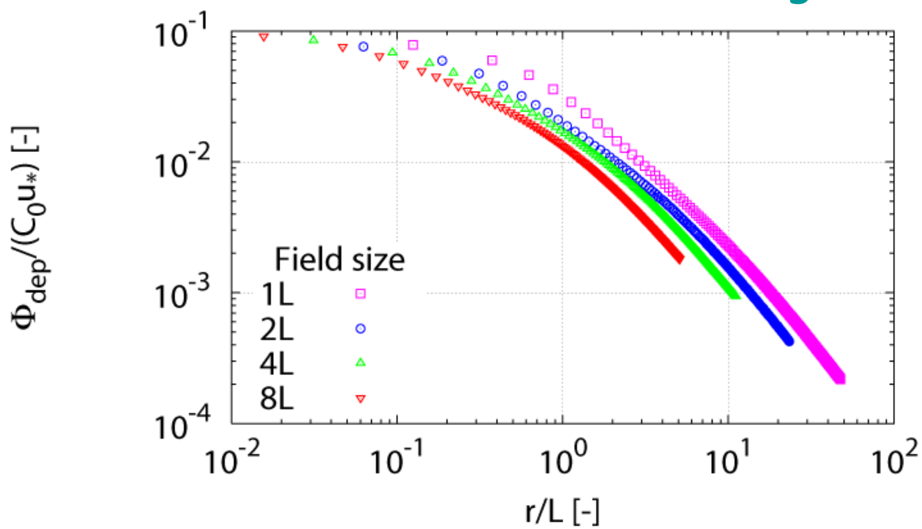
Effect of source field size on pollen dispersion

Simulation for 4 field sizes:

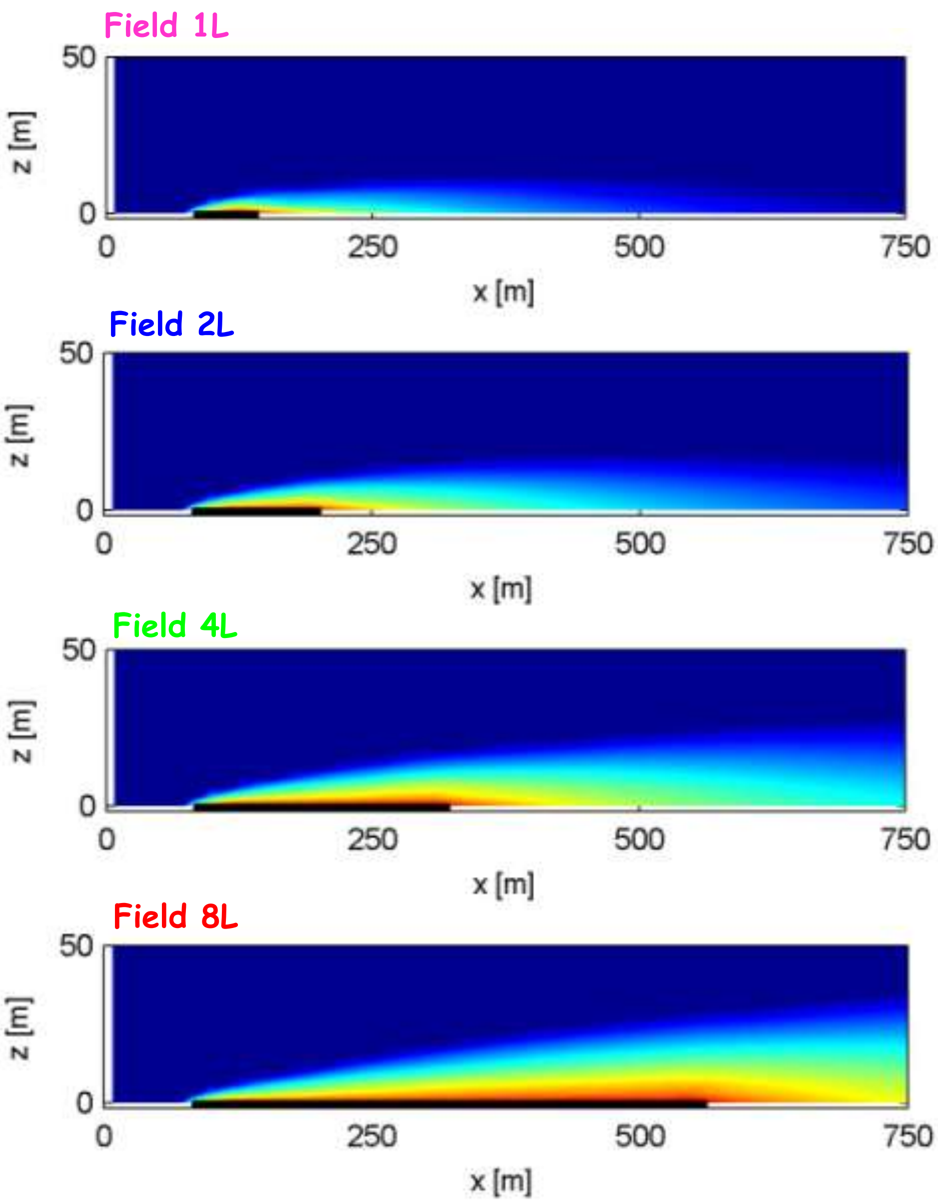




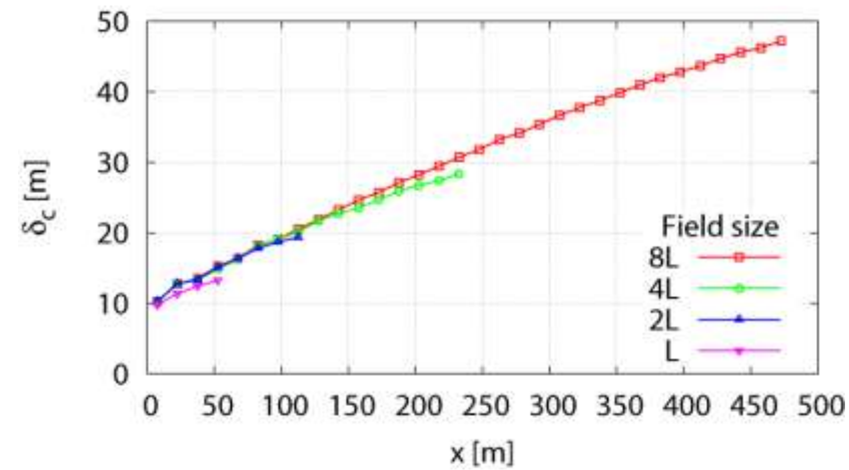
Test of Shaw et al. scaling



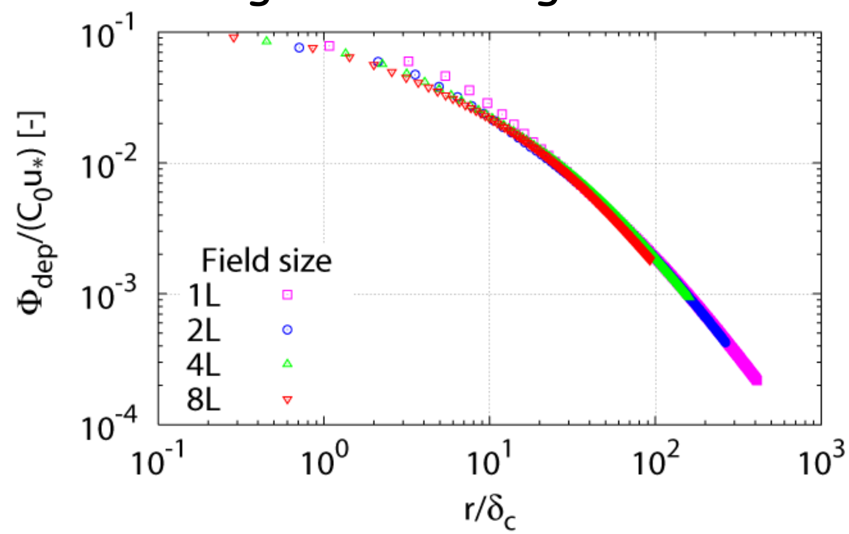
Pollen concentration boundary layer



- Pollen BL growth:



- Scaling with BL height:



Challenges: moving to more realistic conditions

- Moving toward more "realistic" conditions require:

  - Replacing periodic horizontal b.c.'s by inflow from regional/mesoscale models

  - Coupling radiation and dynamics (surface energy balance, clouds, etc.)

  - Resolving vegetated surfaces (instead of modeling trees as roughness)

  - Accounting for topography

  - Resolving urban areas (buildings, etc.)

  - Including surface heterogeneity (resolved and subgrid)

  - …

Thanks!

Equilibrium-stress wall model for ABL

- Using MOS and the equilibrium-stress assumption

  - Determine the kinematic wall stress from resolved velocity:

$$u_* = \left[\frac{\kappa \tilde{u}_h}{\ln\left(\frac{z}{z_o}\right) - \Psi_m\left(\frac{z}{L}\right)}\right]$$

Resolved horizontal velocity at first grid point

$$\tilde{u}_h = \left[(\tilde{u}_1)^2 + (\tilde{u}_2)^2\right]^{1/2}$$

  - Divide stress in components proportional to velocity components:

Streamwise: $\tau_{w,1} = -\rho u_*^2 \left(\frac{\tilde{u}_1}{\tilde{u}_h}\right)$

Spamwise: $\tau_{w,2} = -\rho u_*^2 \left(\frac{\tilde{u}_2}{\tilde{u}_h}\right)$

This is a VERY rough method to enforce boundary conditions!
(We need to do better than that)