Introduction of Particle Image Velocimetry

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Slides largely generated by
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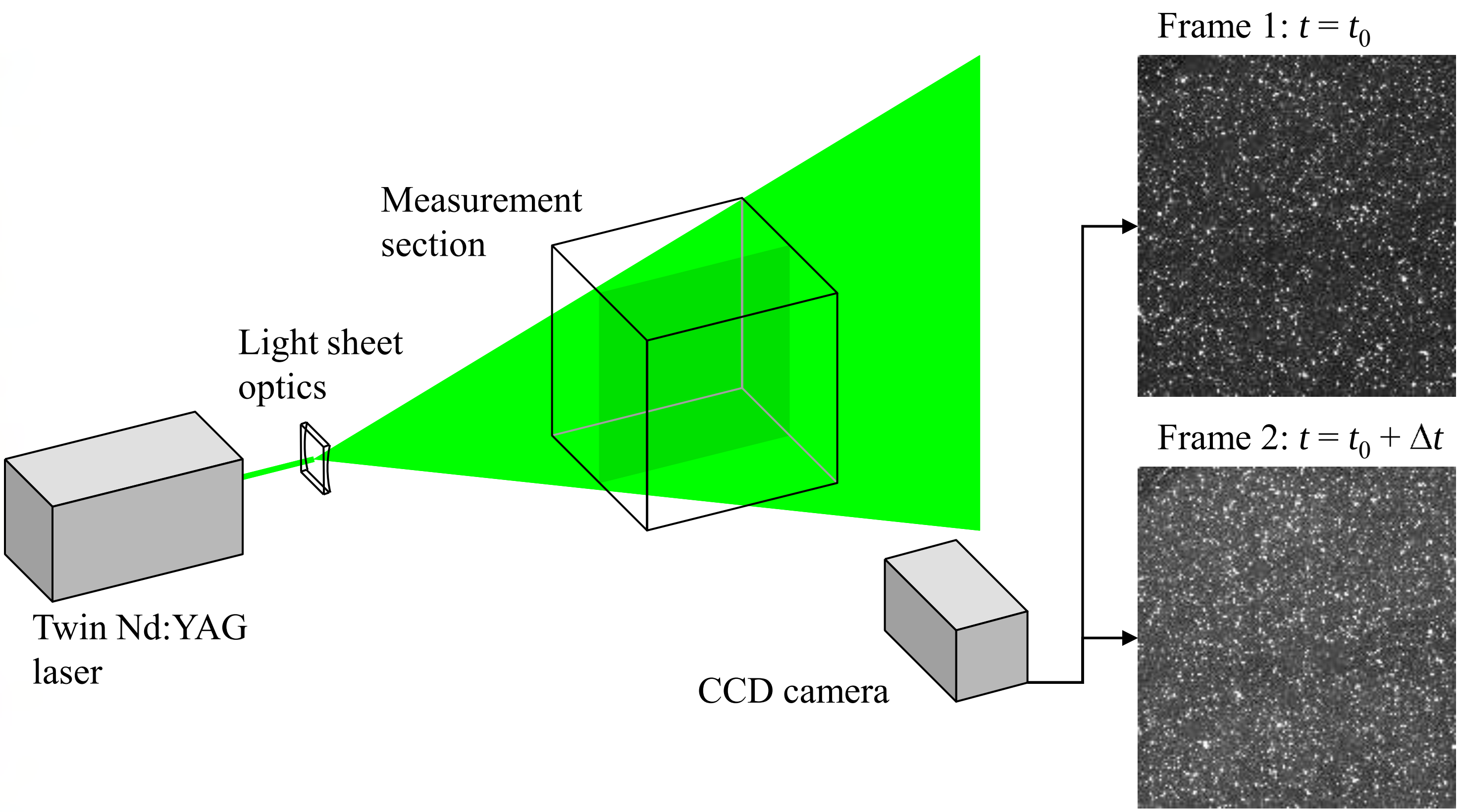
Adapted by K. Kiger

Introduction

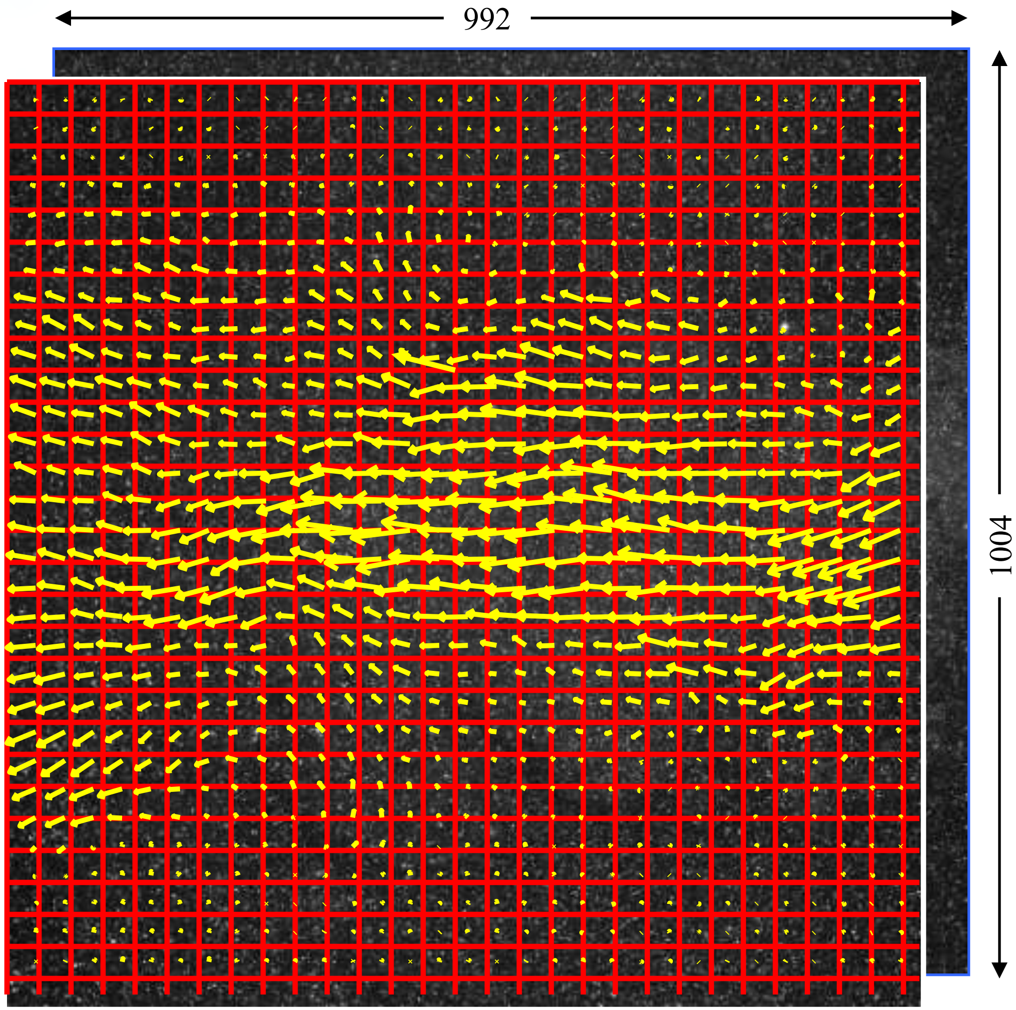
Particle Image Velocimetry (PIV):

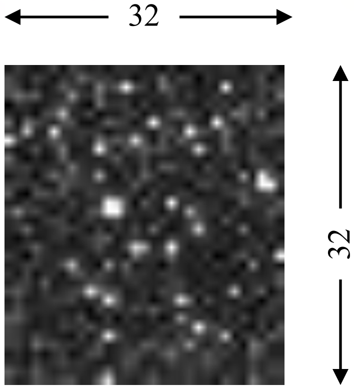
Imaging of tracer particles, calculate displacement: local fluid velocity



Introduction

Particle Image Velocimetry (PIV)





- divide image pair in *interrogation regions*
- small region:
  ~ uniform motion
- compute displacement
- ***repeat !!!***

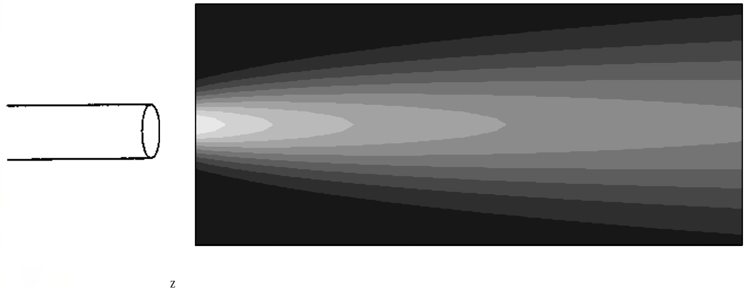
Introduction

Particle Image Velocimetry (PIV):

Instantaneous measurement of 2 components in a plane

conventional methods
(HWA, LDV)

- single-point measurement
- traversing of flow domain
- time consuming
- only turbulence statistics



particle image velocimetry

- whole-field method
- non-intrusive (seeding
- instantaneous flow field

Introduction

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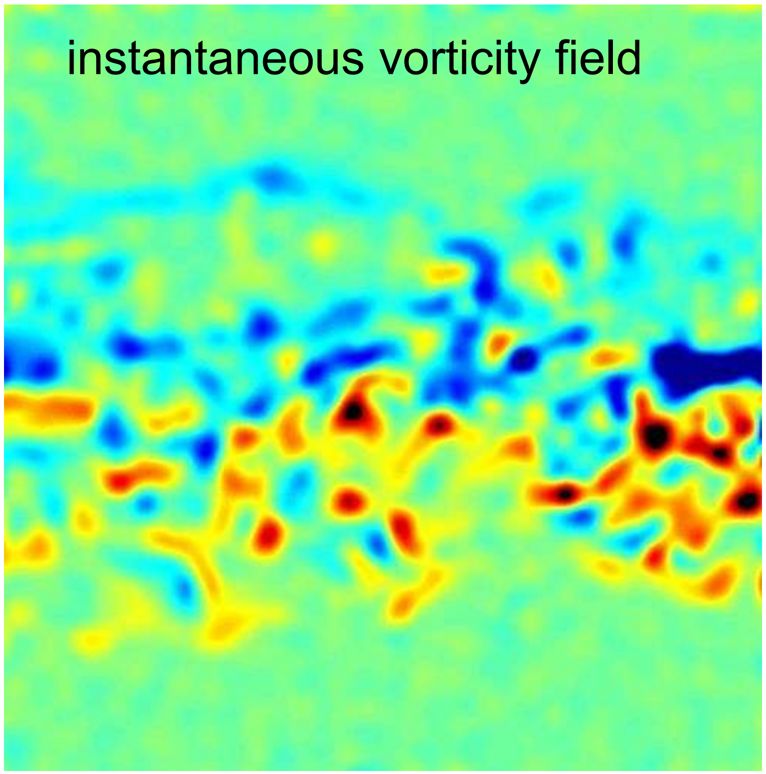
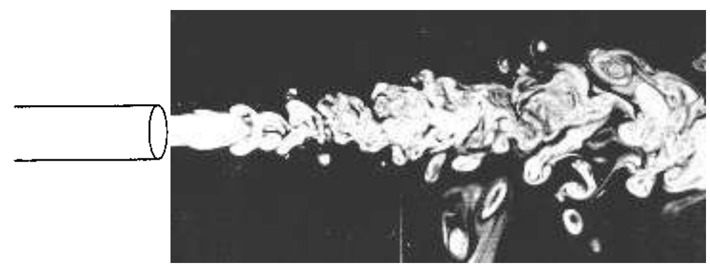
Instantaneous measurement of 2 components in a plane

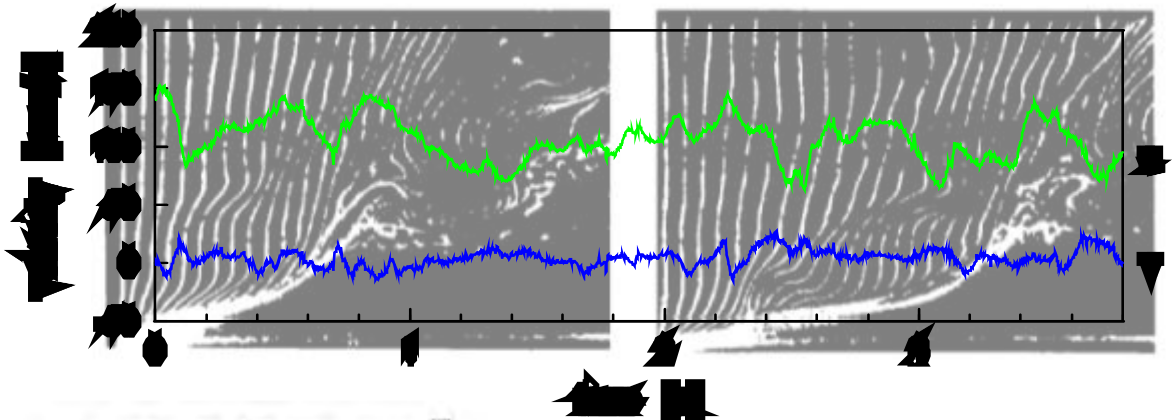


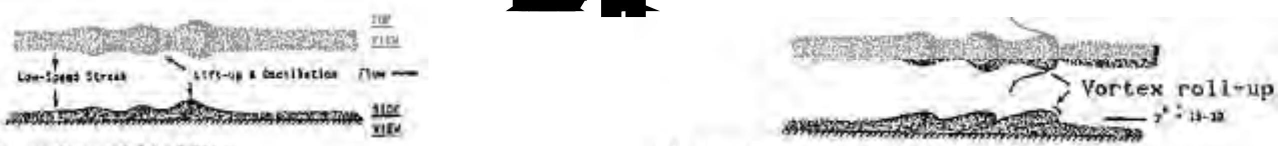
particle image velocimetry

- whole-field method
- non-intrusive (seeding
- instantaneous flow field



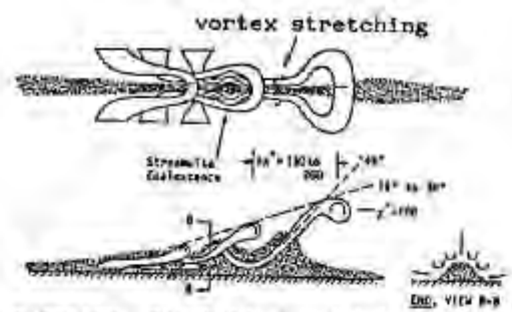
Example: coherent structures

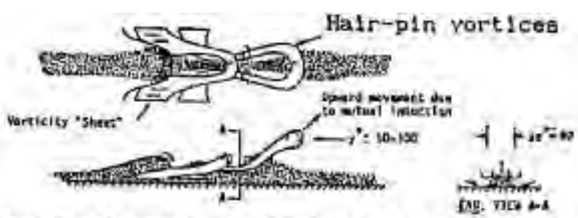




a) Lift-up and oscillation

b) initiation of vortex roll-up

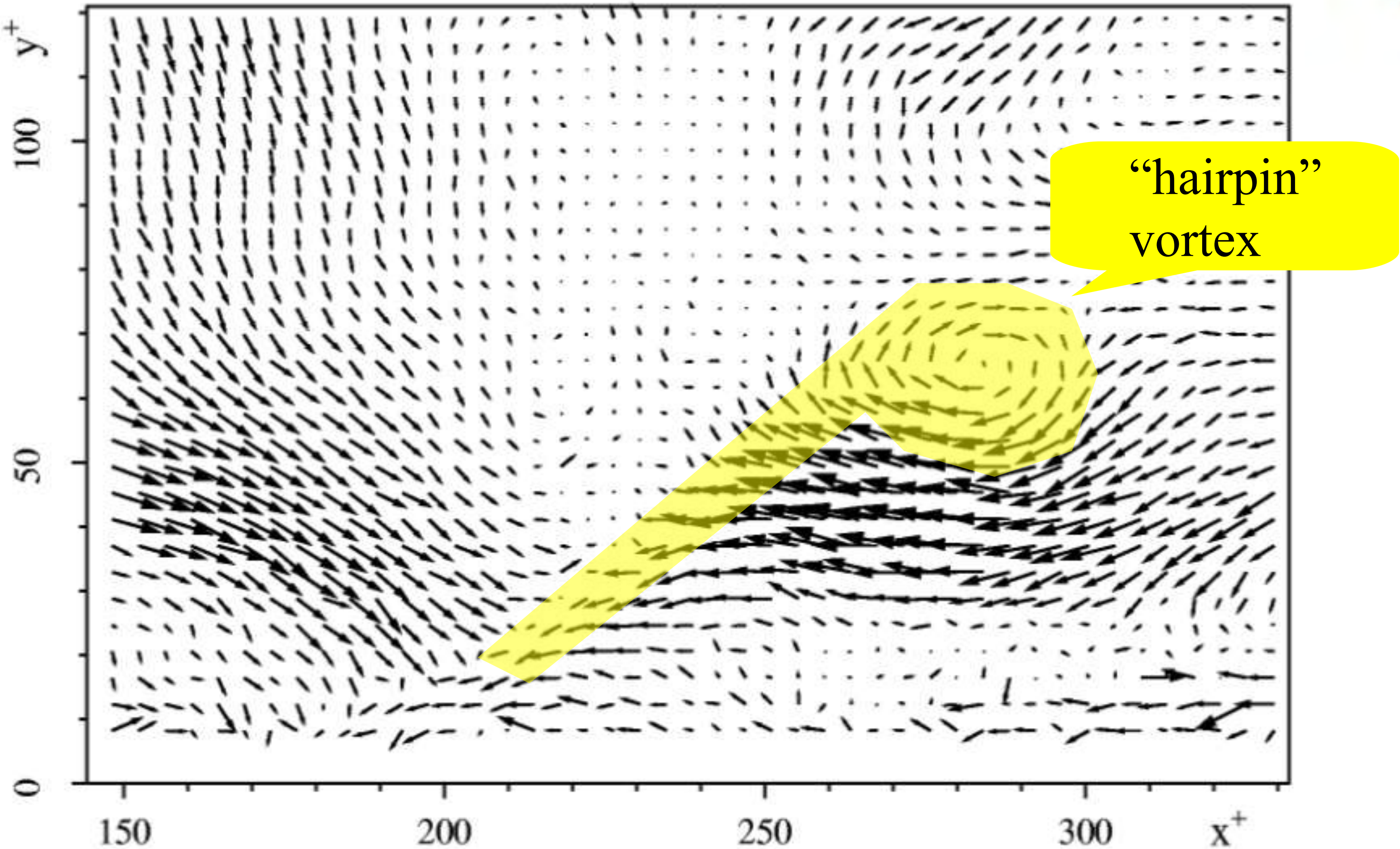




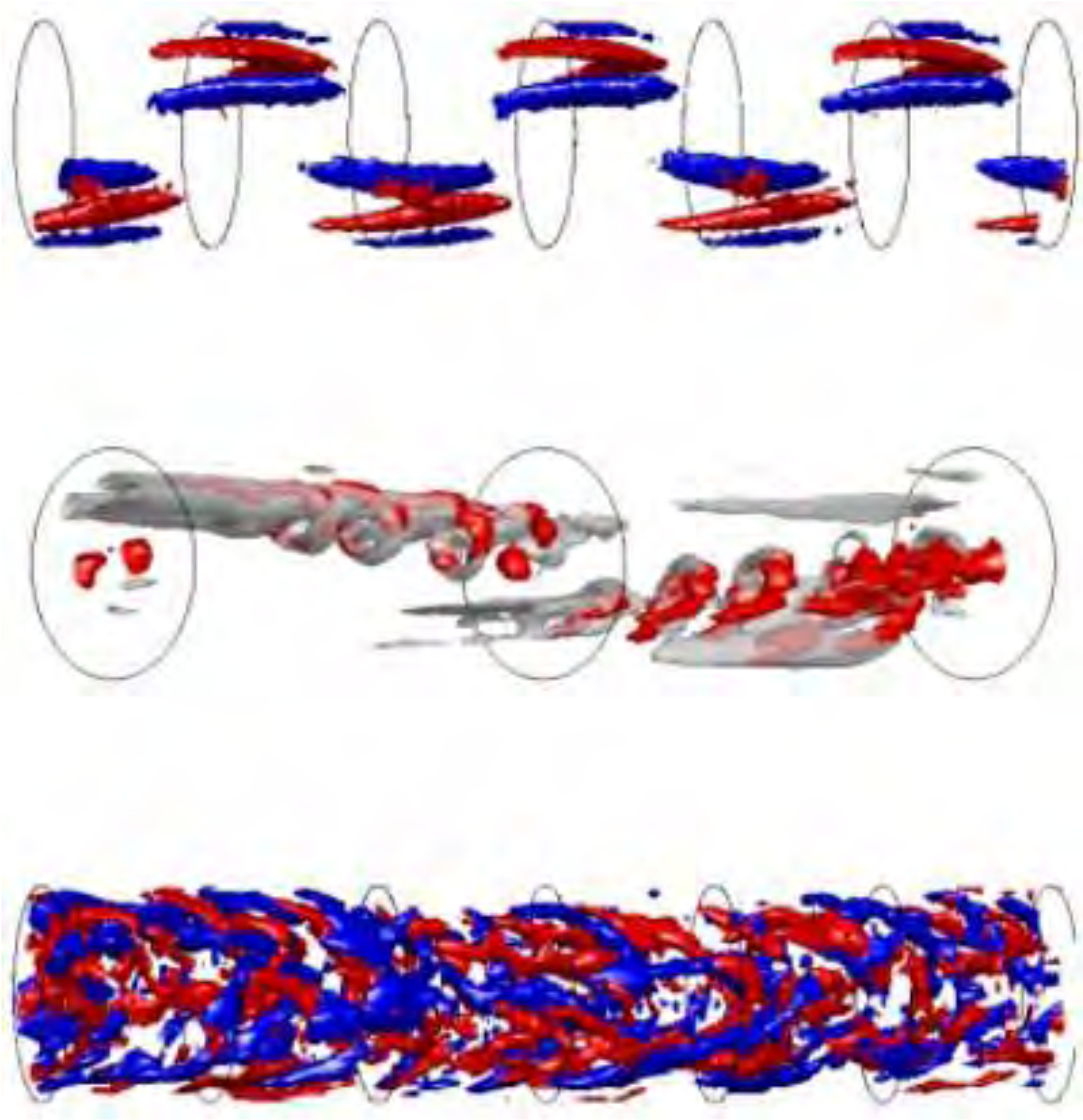
c) vortex development: amplification and concentration

d) vortex ejection, stretching and interaction

Example: coherent structures

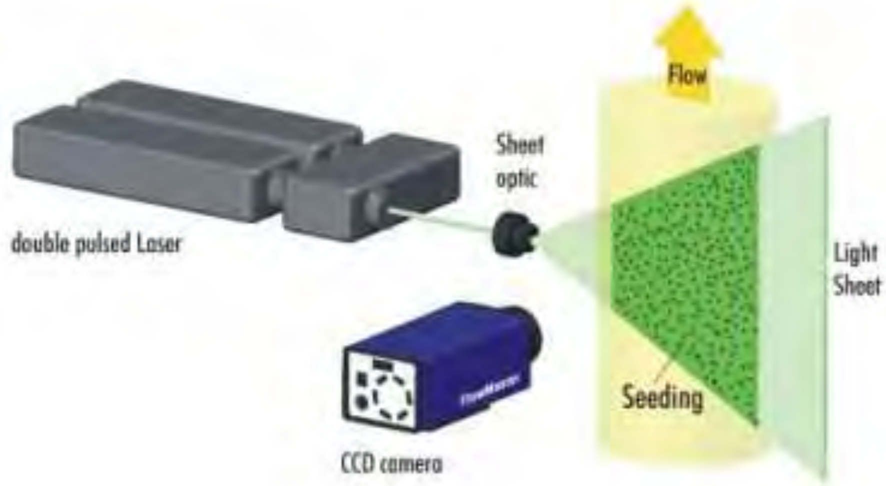


Example: coherent structures



Van Doorne, *et al.*

Overview



PIV components:

- tracer particles
- light source
- light sheet optics
- camera

Hardware (imaging)

- measurement settings

- interrogation
- post-processing

Software (image analysis)

Tracer particles

Assumptions:

- homogeneously distributed
- follow flow perfectly
- uniform displacement within interrogation region

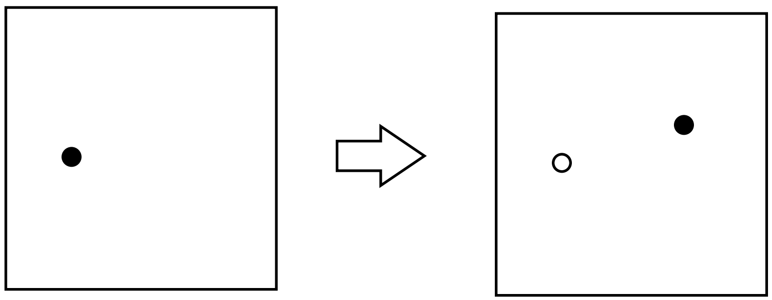
Criteria:

-easily visible
-particles should not influence fluid flow!

↓

small, volume fraction < 10^{-4}

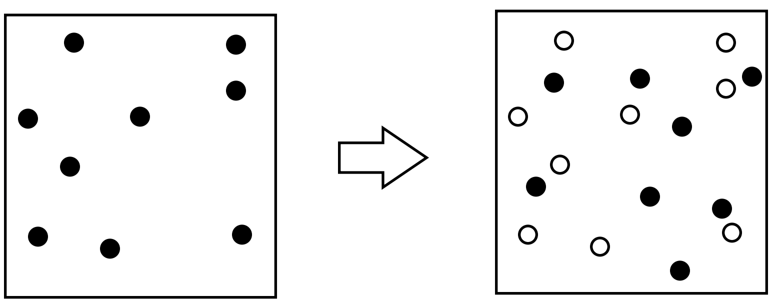
Image density



low image density

$N_I << 1$

particle tracking velocimetry



high image density

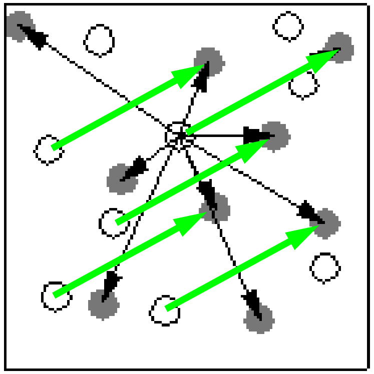
$N_I >> 1$

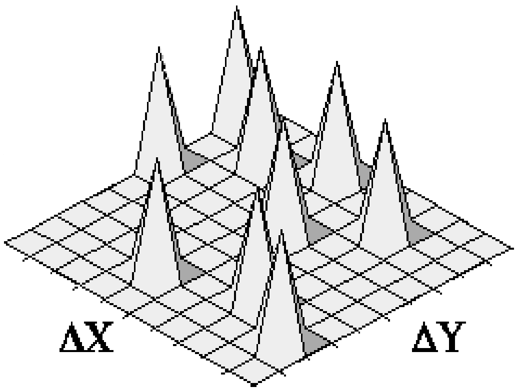
particle image velocimetry

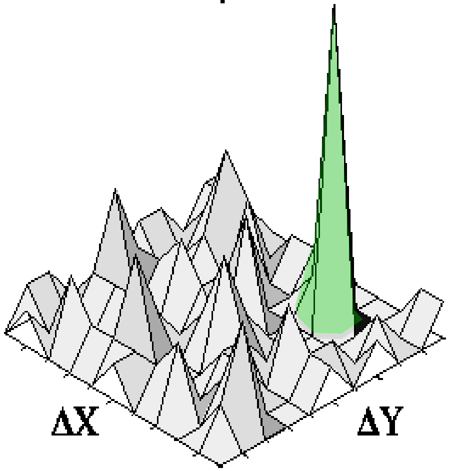
Evaluation at higher density

High N_I : no longer possible/desirable to follow individual tracer particles

Possible „matches' Sum of all possibilities







Particle can be matched with a number of candidates

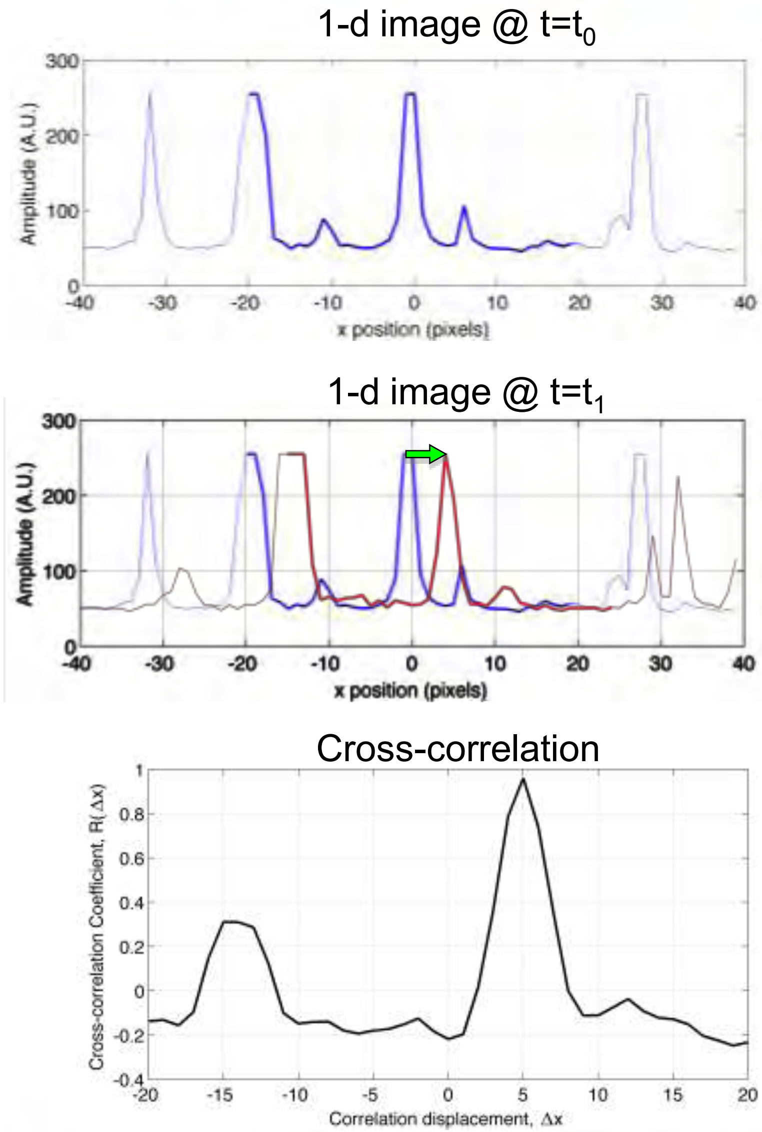
Repeat process for other particles, sum up: "wrong" combinations will lead to noise, but "true" displacement will dominate

Statistical estimate of particle motion

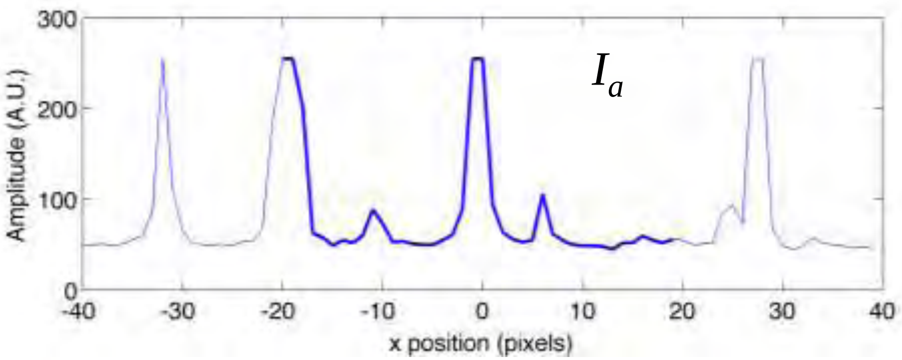
- Statistical correlations used to find average particle displacement

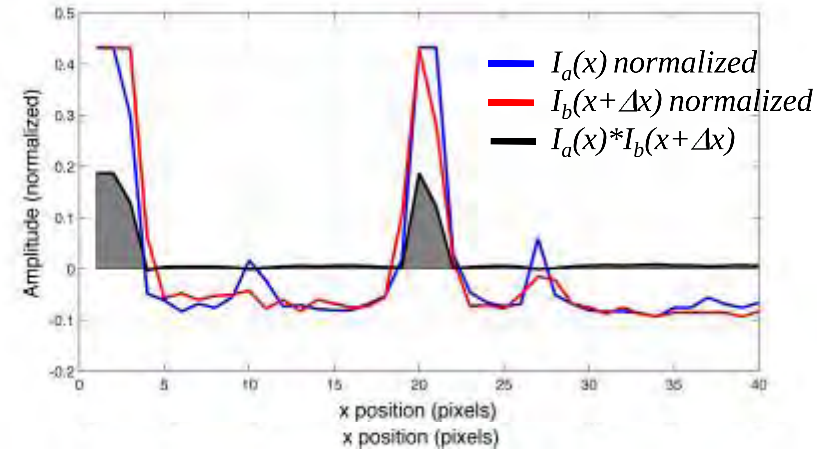
$$R(i,j)=\frac{\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}\left(I_a(k,l)-\bar{I}_a\right)\left(I_b(k+i,l+j)-\bar{I}_b\right)}{\left[\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}\left(I_a(k,l)-\bar{I}_a\right)^2\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}\left(I_b(k+i,l+j)-\bar{I}_b\right)^2\right]^{\frac{1}{2}}}$$

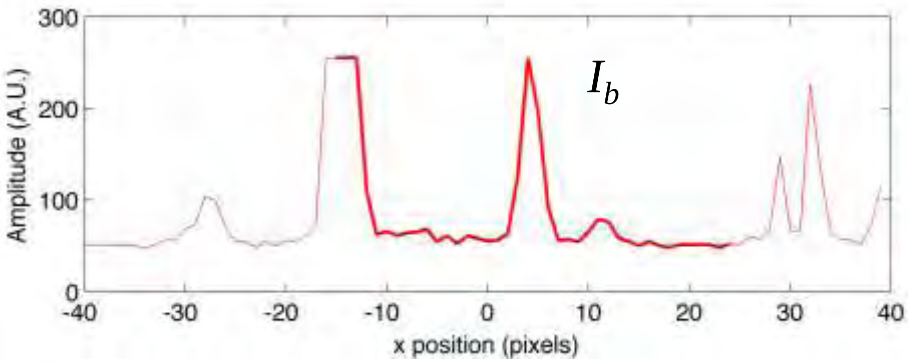
$$\bar{I}_a=\frac{1}{B_xB_y}\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}I_a(k,l)$$

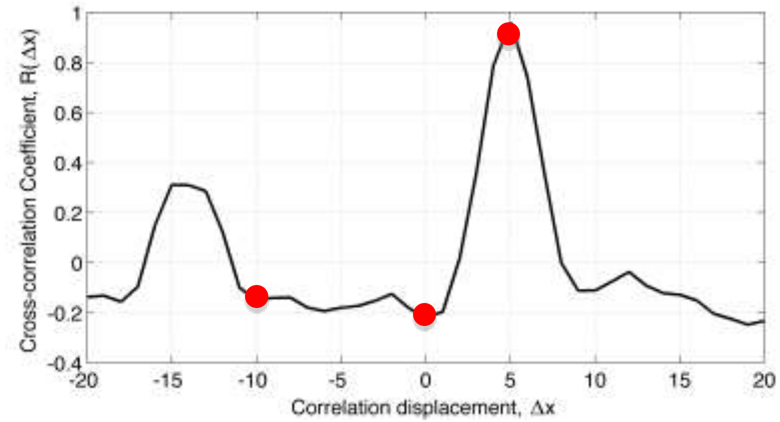


1-D cross-correlation example









$$R(i)=\frac{\sum_{k=1}^{B_x}\left(I_a(k)-\bar{I}_a\right)\left(I_b(k+i)-\bar{I}_b\right)}{\left[\sum_{k=1}^{B_x}\left(I_a(k)-\bar{I}_a\right)^2\sum_{k=1}^{B_x}\left(I_b(k+i)-\bar{I}_b\right)^2\right]^{\frac{1}{2}}}$$

Finding the maximum displacement

-Shift 2nd window with respect to the first

Typically 16x16 or 32x32 pixels

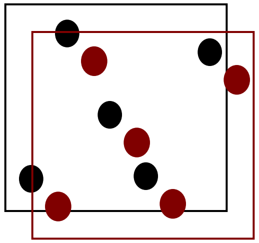
- Calculate "match"

Good indicator:

$$R(i,j)=\frac{\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}\left(I_a(k,l)-\bar{I}_a\right)\left(I_b(k+i,l+j)-\bar{I}_b\right)}{\left[\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}\left(I_a(k,l)-\bar{I}_a\right)^2\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}\left(I_b(k+i,l+j)-\bar{I}_b\right)^2\right]^{\frac{1}{2}}}$$

- Repeat to find best estimate

$$\bar{I}_a=\frac{1}{B_xB_y}\sum_{k=1}^{B_x}\sum_{l=1}^{B_y}I_a(k,l)$$



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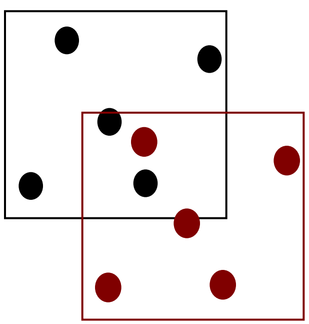

Bad match: sum of product of intensities low

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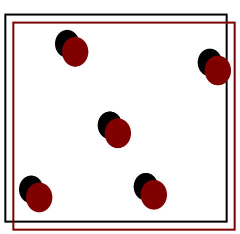


Good match: sum of product of intensities high

◆Can be implemented as 2D FFT for digitized data

- Impose periodic conditions on interrogation region…causes bias error if not treated properly.

Cross-correlation

This "shifting" method can formally be expressed as a cross-correlation:

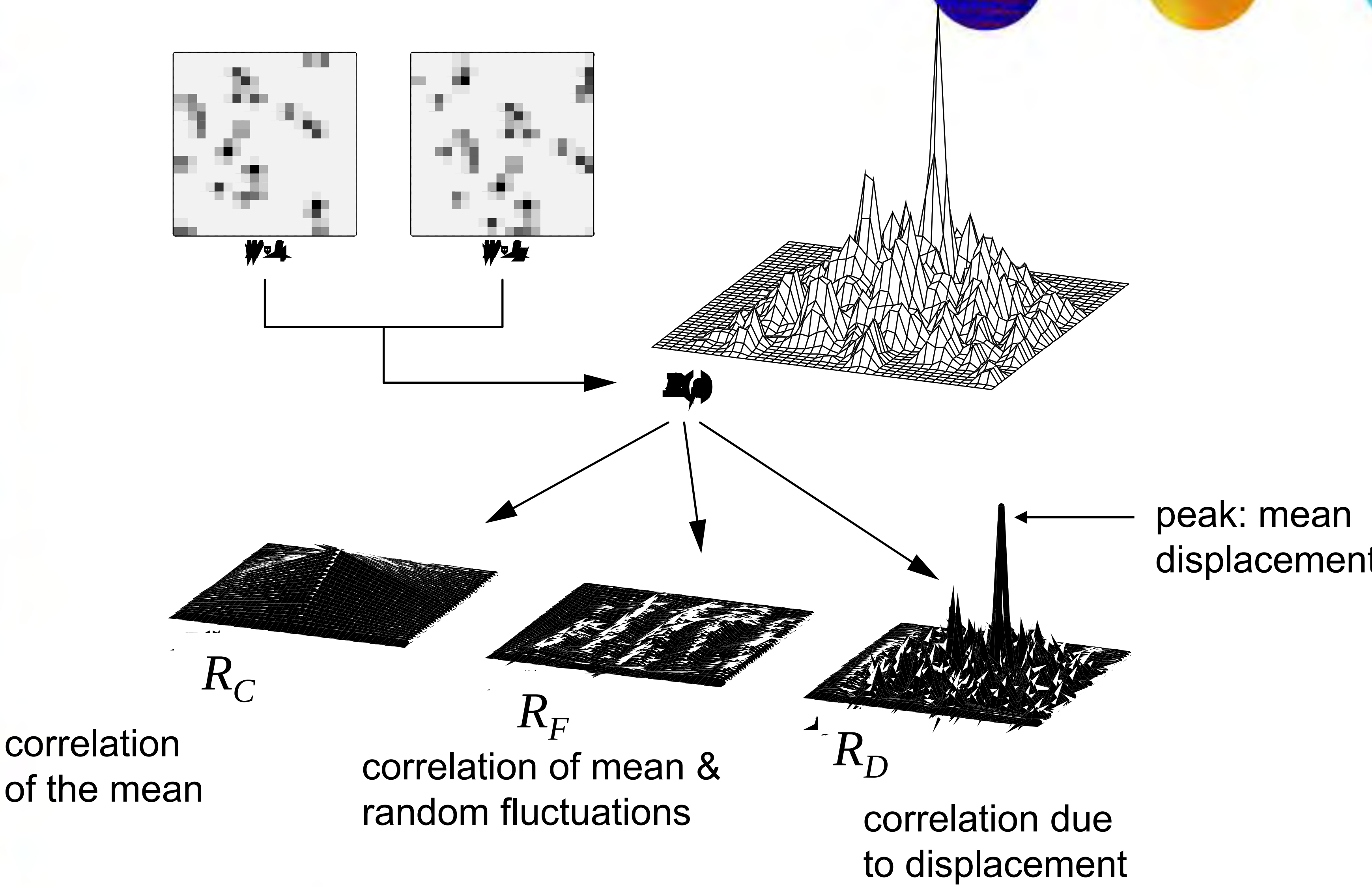
$$R(\mathbf{s}) = \int I_1(\mathbf{x})\, I_2(\mathbf{x}+\mathbf{s})\, d\mathbf{x}$$

- I_1 and I_2 are interrogation areas (sub-windows) of the total frames
- x is interrogation location
- s is the shift between the images

"Backbone" of PIV:

-cross-correlation of interrogation areas
-find location of displacement peak

Cross-correlation

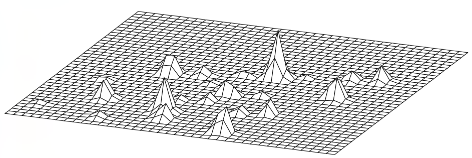


Influence of N_I

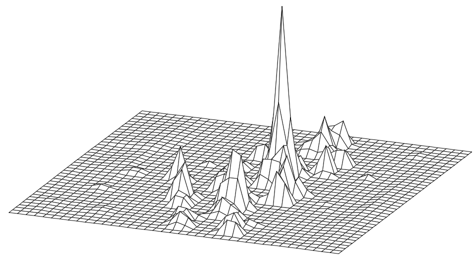
$$R_D(\mathbf{s}_D) \sim N_I \quad \Rightarrow \quad N_I = \frac{C\Delta z_0}{M_0^2} D_I^2$$

C	particle concentration
Δz_0	light sheet thickness
D_I	int. area size
M_0	magnification

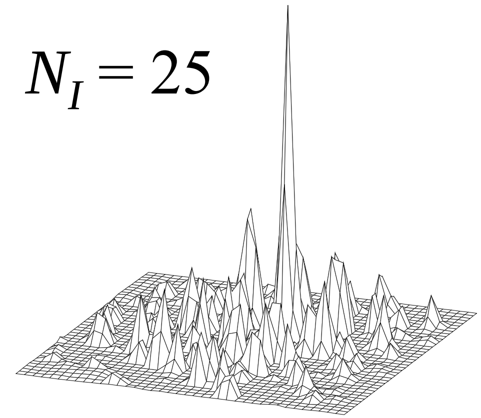
$N_I = 5$



$N_I = 10$



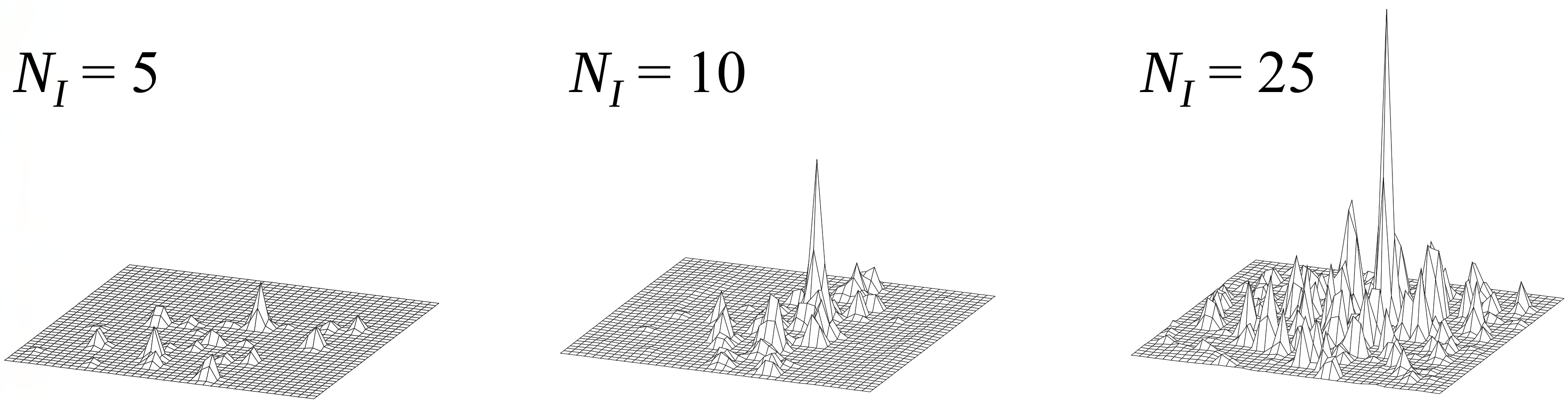
$N_I = 25$



More particles: better signal-to-noise ratio

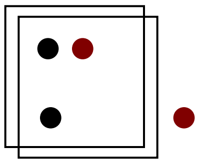
Unambiguous detection of peak from noise:
N_I=10 (average), minimum of 4 per area in 95% of areas
(number of tracer particles is a Poisson distribution)

Influence of N_I

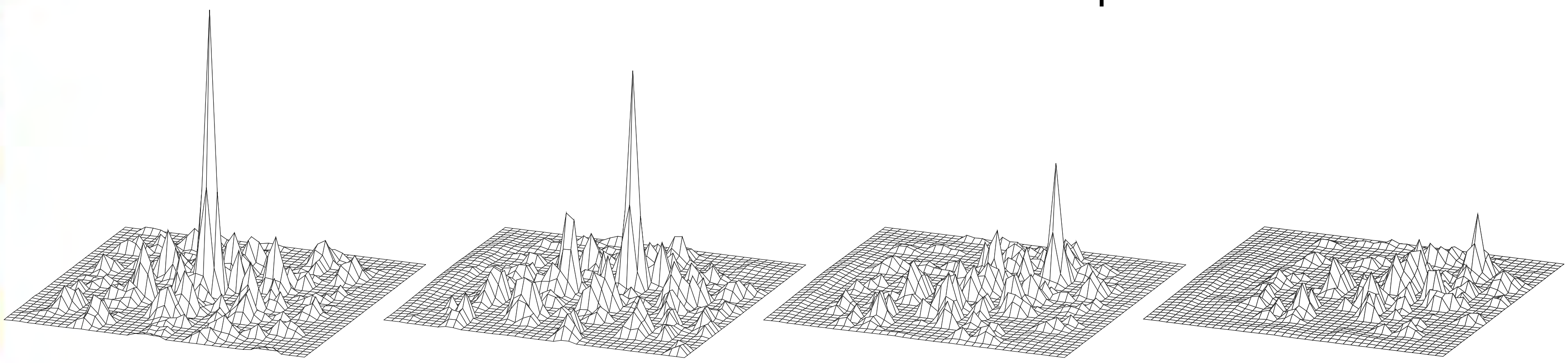


PTV: 1 particle used for velocity estimate; error *e*
PIV: error ~ *e*/sqrt(N_I)

Influence of in-plane displacement



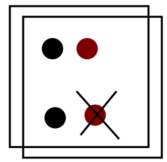
X,Y-Displacement
<
quarter of window size



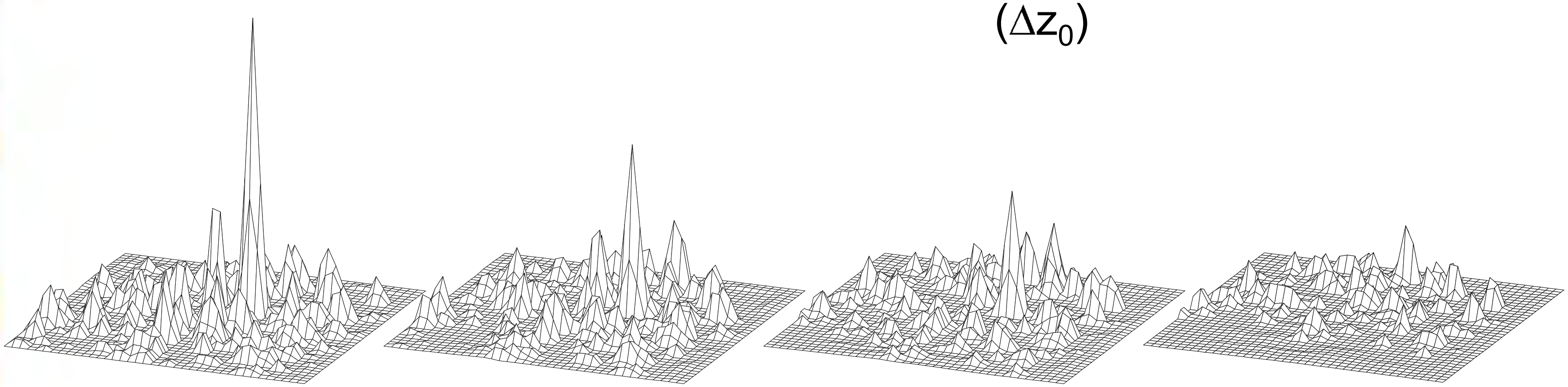
$\Delta X / D_I = 0.00$	0.28	0.56	0.85
$F_I = 1.00$	0.64	0.36	0.16

$$R_D(\mathbf{s}_D) \sim N_I F_I \Rightarrow F_I(\Delta X, \Delta Y) = \left(1 - \frac{|\Delta X|}{D_I}\right)\left(1 - \frac{|\Delta Y|}{D_I}\right)$$

Influence of out-of-plane displacement



Z-Displacement
<
quarter of light sheet thickness
(Δz_0)

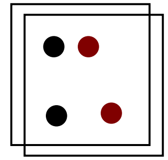


$\Delta Z / \Delta z_0 =$	0.00	0.25	0.50	0.75
$F_O =$	1.00	0.75	0.50	0.25

$$R_D(\mathbf{s}_D) \sim N_I F_I F_O \quad \Rightarrow \quad F_O(\Delta z) = 1 - \frac{|\Delta z|}{\Delta z_0}$$

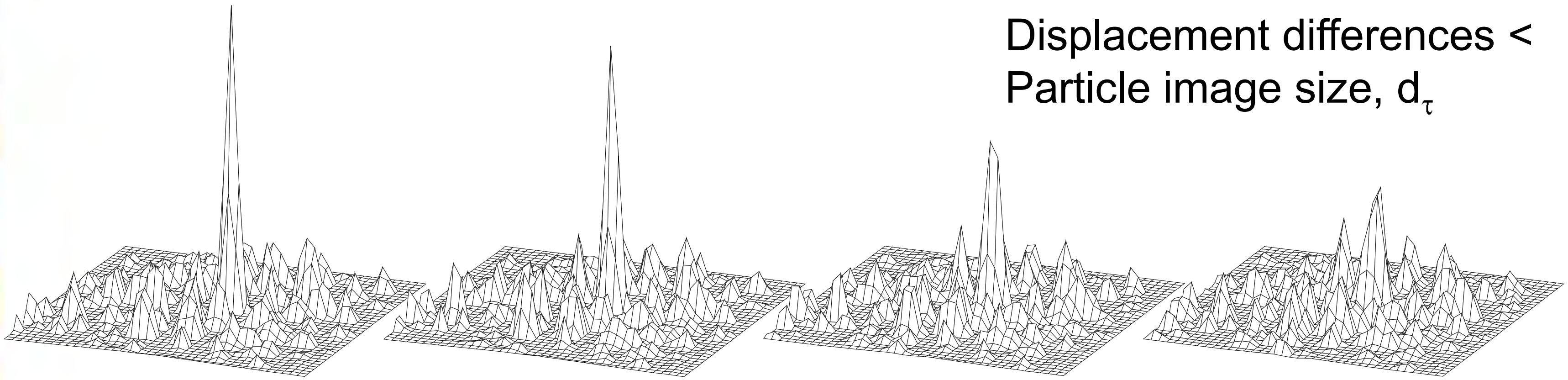
Influence of gradients

$$a \equiv M_0 |\Delta u| \Delta t$$



Displacement differences < 3-5% of int. area size, D_I

Displacement differences < Particle image size, d_τ



$a / D_I = 0.00$	0.05	0.10	0.15
$a / d_\tau = 0.00$	0.50	1.00	1.50

$$R_D(\mathbf{s}_D) \sim N_I F_I F_O F_\Delta \quad \Rightarrow \quad F_\Delta(a) \cong \exp(-a^2 / d_\tau^2)$$

R.D. Keane & R.J. Adrian

PIV "Design rules"

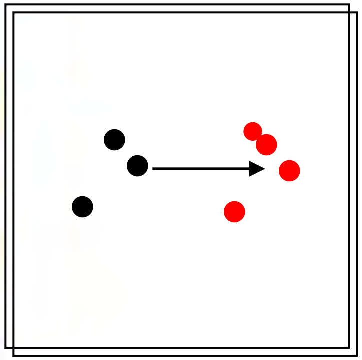
- image density $N_I > 10$
- in-plane motion $|\Delta X| < \frac{1}{4} D_I$
- out-of-plane motion $|\Delta z| < \frac{1}{4} \Delta z_0$
- spatial gradients $M_0|\Delta u|\Delta t < d_\tau$

Obtained by Keane & Adrian (1993) using synthetic data

Window shifting

- in-plane motion $|\Delta X| < \frac{1}{4} D_I$

strongly limits dynamic range of PIV

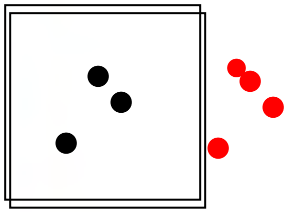


large window size: too much spatial averaging

Window shifting

- in-plane motion $|\Delta X| < \frac{1}{4} D_I$

strongly limits dynamic range of PIV

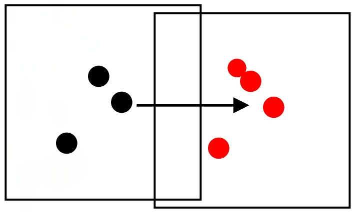


small window size: too much in-plane pair loss

Window shifting

- in-plane motion $|\Delta X| < \frac{1}{4} D_I$

strongly limits dynamic range of PIV



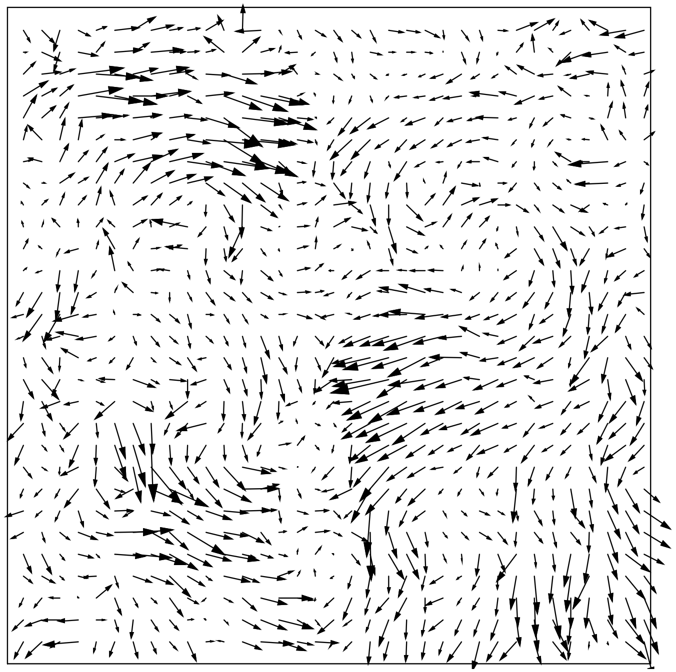
Multi-pass approach:
start with large windows,
use this result as „pre-shift'
for smaller windows…

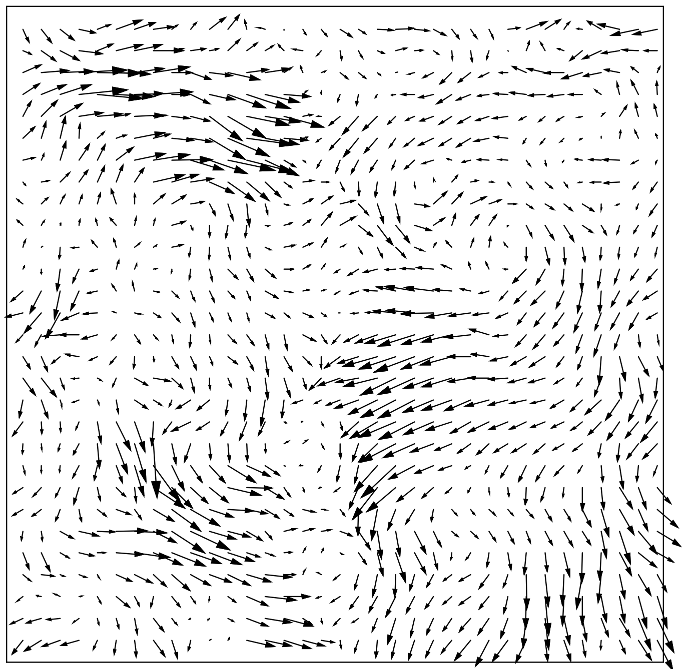
No more in-plane pair loss limitations!

Window shifting: Example

Grid turbulence

fixed windows	matched windows



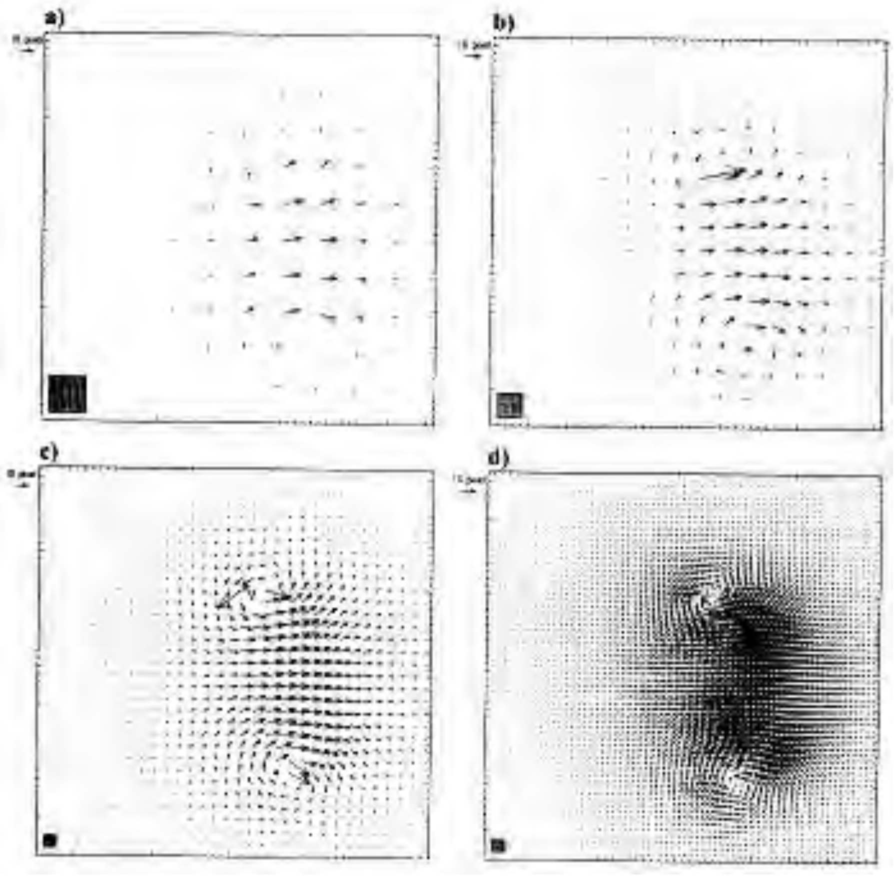


windows at same location

windows at 7px „downstream'

Window shifting: Example

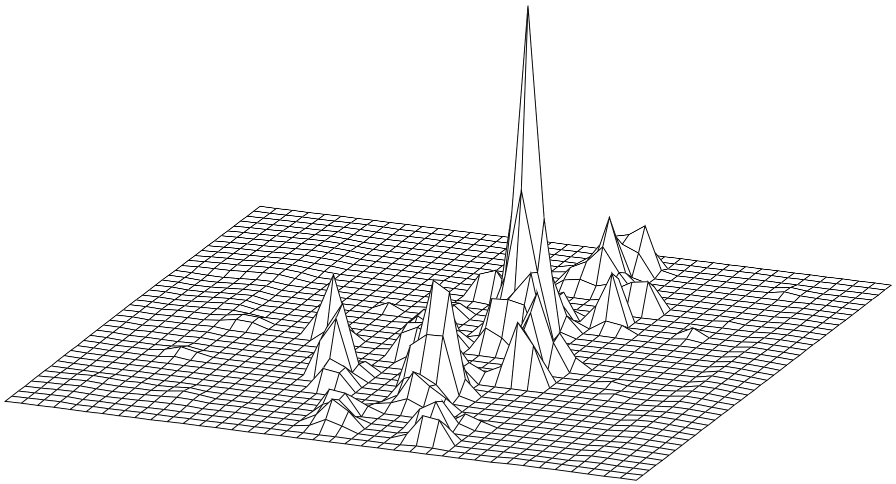
Vortex ring, decreasing window sizes



Raffel,
Willert and
Kompenhans

Sub-pixel accuracy

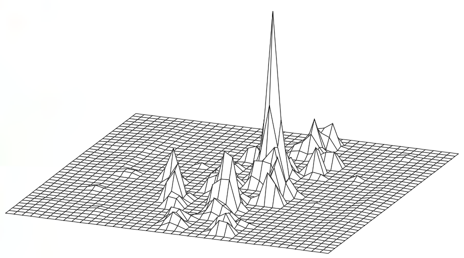
Maximum in the correlation plane: single-pixel resolution of displacement?

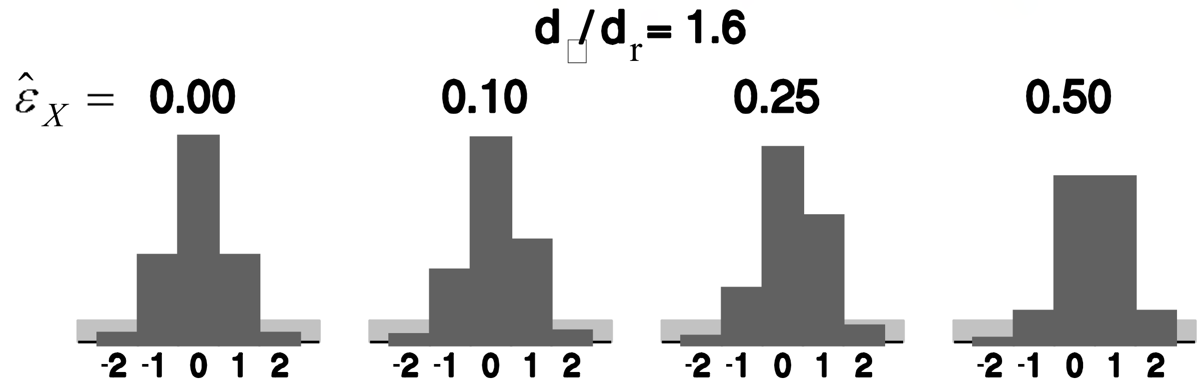


But the peak contains a lot more information!

Gaussian particle images → Gaussian correlation peak (but smeared)

Sub-pixel accuracy





Fractional displacement can be obtained using the distribution of gray values around maximum

three-point estimators

- **peak centroid**

$$\varepsilon = \frac{R_{+1} - R_{-1}}{R_{-1} + R_0 + R_{+1}}$$

- **parabolic peak fit**

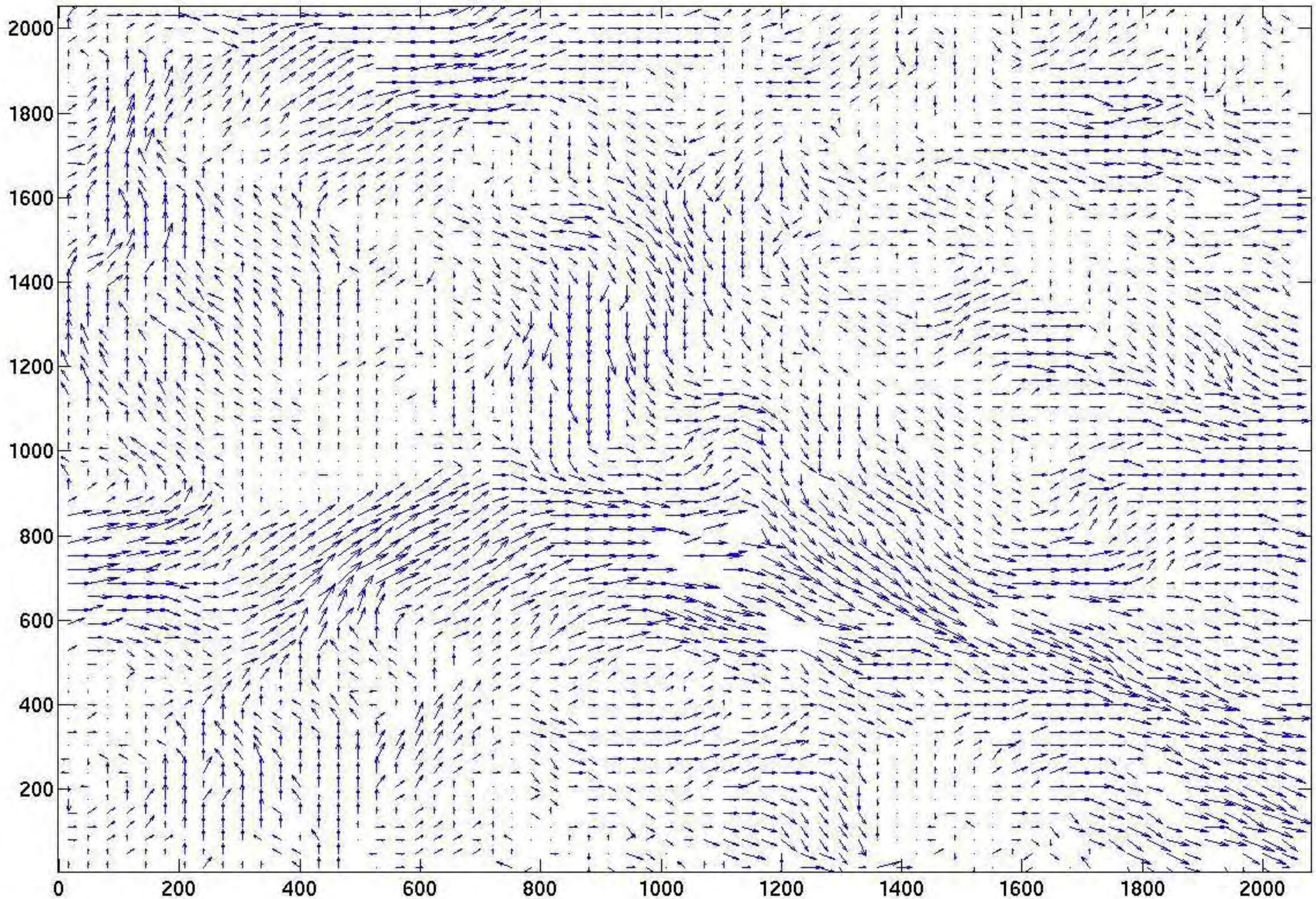
$$\varepsilon = \frac{R_{-1} - R_{+1}}{2\left(R_{-1} + R_{+1} - 2R_0\right)}$$

- **Gaussian peak fit**

$$\varepsilon = \frac{\ln R_{-1} - \ln R_{+1}}{2\left(\ln R_{-1} + \ln R_{+1} - 2\ln R_0\right)}$$

$$\varepsilon \propto \frac{\text{balance}}{\text{normalization}}$$

Peak locking

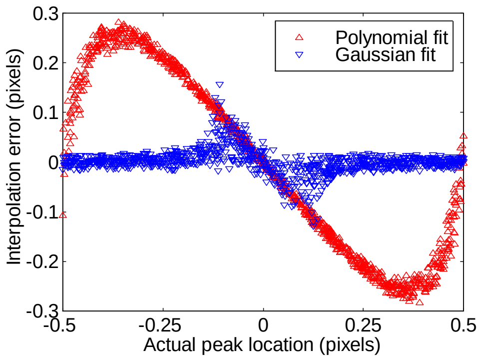


“zig-zag” structure, sudden “kinks” in the flow

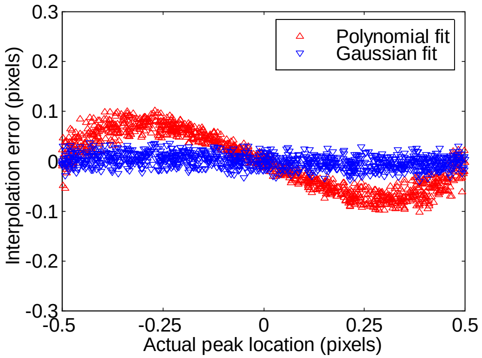
Sub-pixel Interpolation Errors

- **Accuracy depends on:**
  - particle image size
  - noise in data (seeding density, camera noise)
  - shear rate
- **Can exhibit "peak locking"**
  - Interpolation of peak is biased towards a symmetric data distribution (Integer and 1/2 integer peak locations)
  - Polynomials exhibit strong locking when particle diameter is small
  - Gaussian is most commonly used
  - Splines are very robust, but expensive to calculate
- See *Particle Image Velocimetry*, by Raffel, Willert, and Kompenhans, Springer-Verlag, 1998.

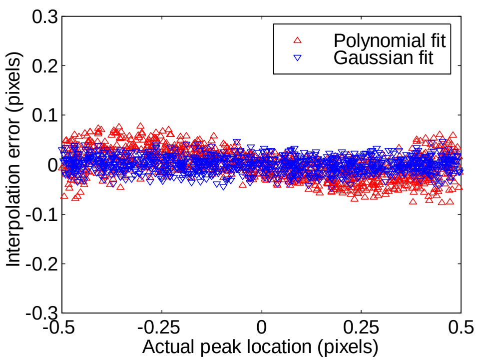
Pixel Radius



R = 0.5



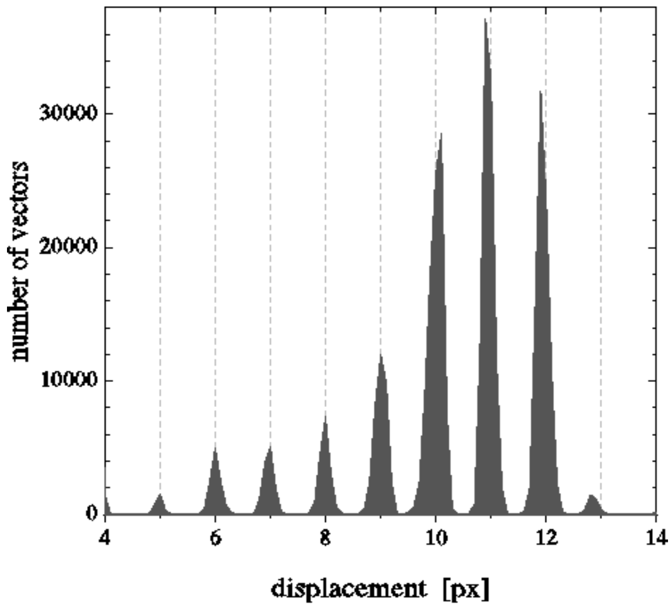
R = 1.0



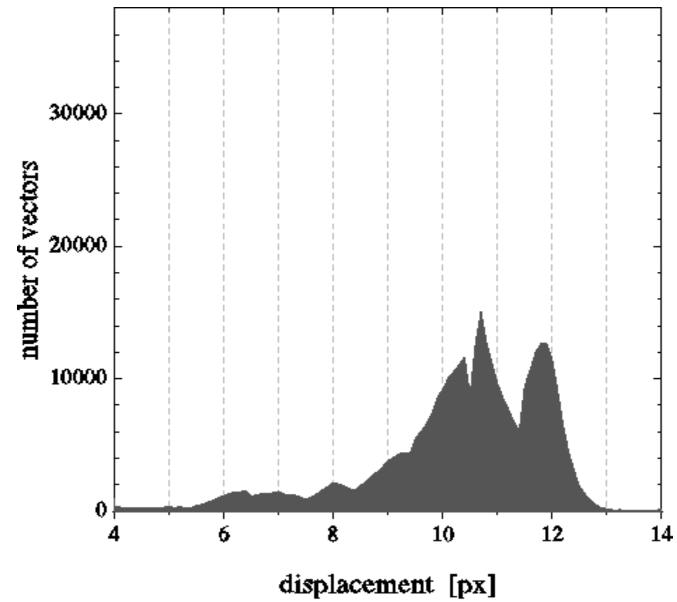
R = 1.5

Peak locking

Histogram of velocities in a turbulent flow



centroid



Gaussian peak fit

Even with Gaussian peak fit:

particle image size too small → peak locking

(Consider a „point particle’ sampled by discrete pixels)

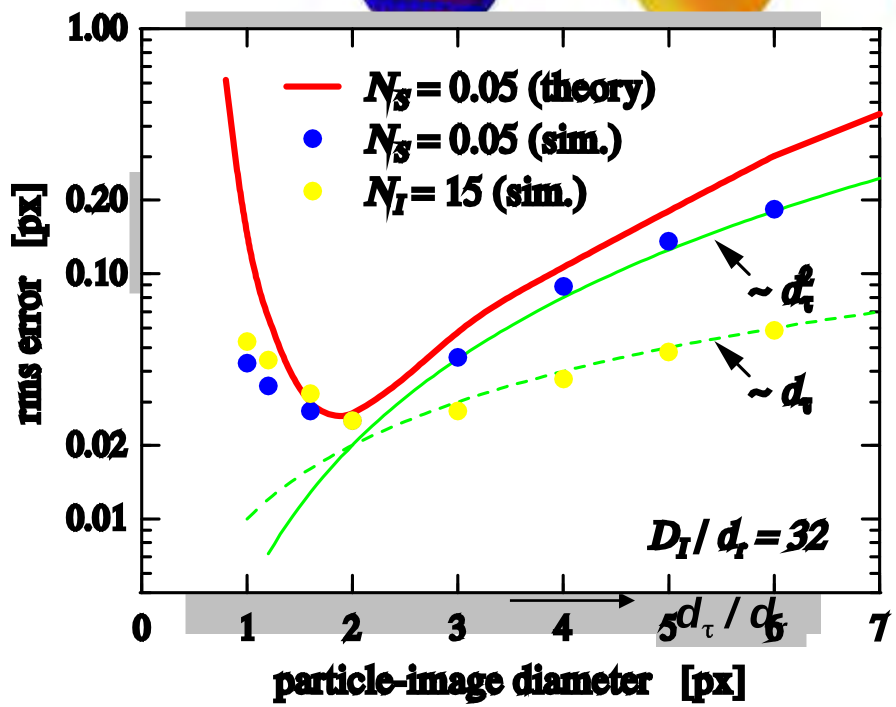
Sub-pixel accuracy

optimal resolution:

particle image size: ~2 px

Smaller: particle no longer resolved

Larger: random noise increase



Theoretical: 0.01 – 0.05 px

In practice 0.05-0.1 px

“three-point” estimators:

Peak centroid

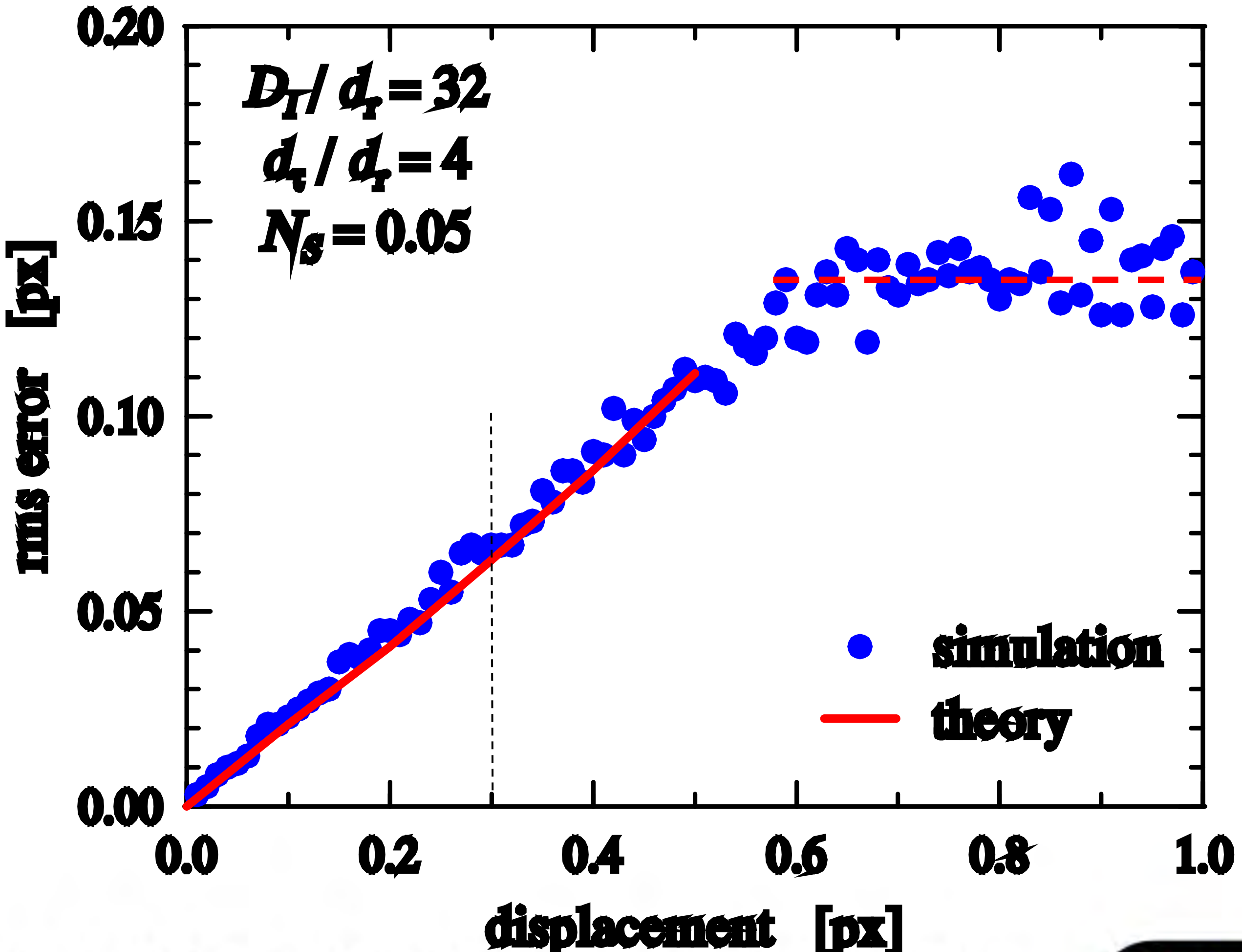
Parabolic peak fit

Gaussian peak fit

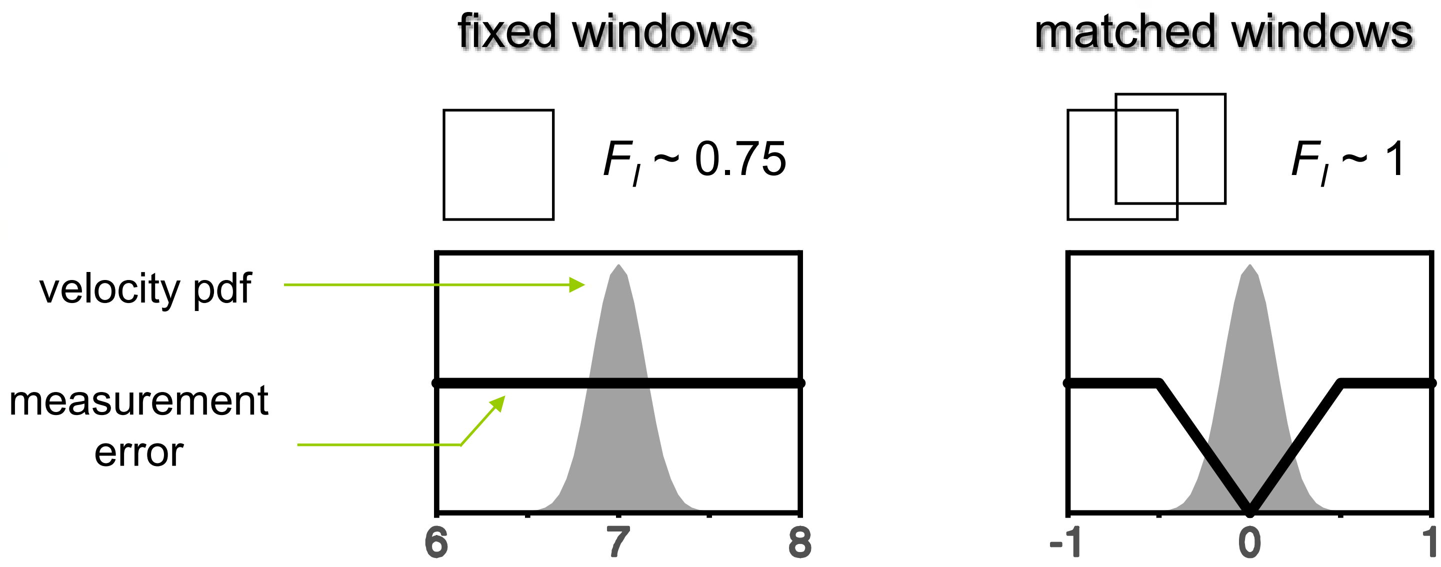
...

Main difference: sensitivity to “peak locking” or “pixel lock-in”, bias towards integer displacements

displacement measurement error



window matching

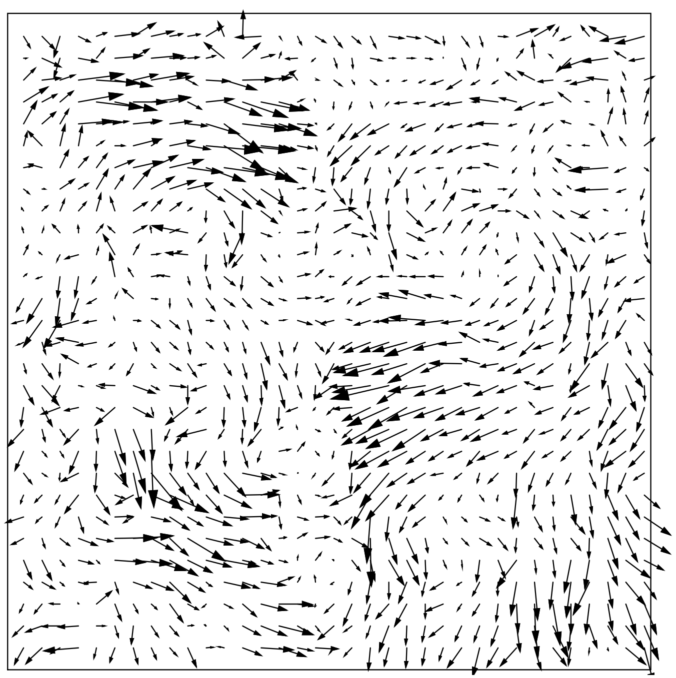


fixed windows		matched windows
u'^2	signal	u'^2
C^2	noise	$4C^2u'^2$
u'^2 / C^2	SNR	$1 / 4C^2$

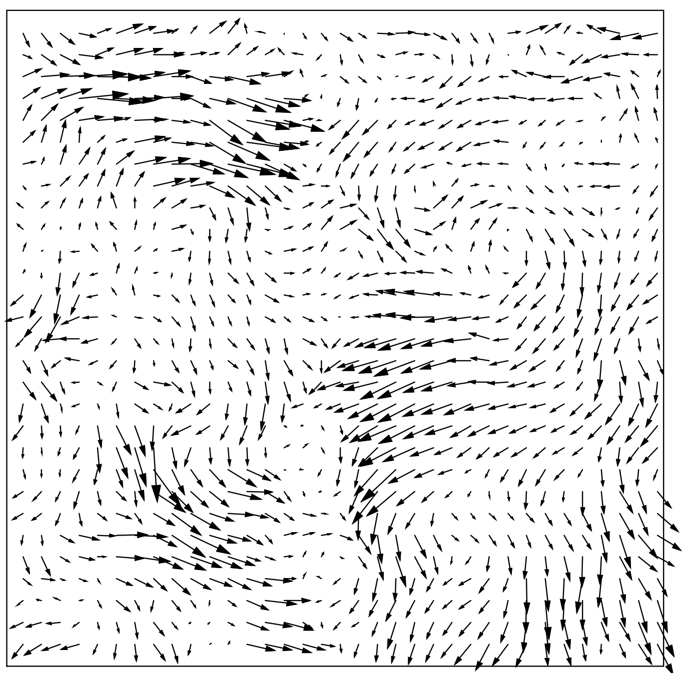
application example: grid-generated turbulence

ΔX = 7 px u'/U = 2.5%

fixed windows

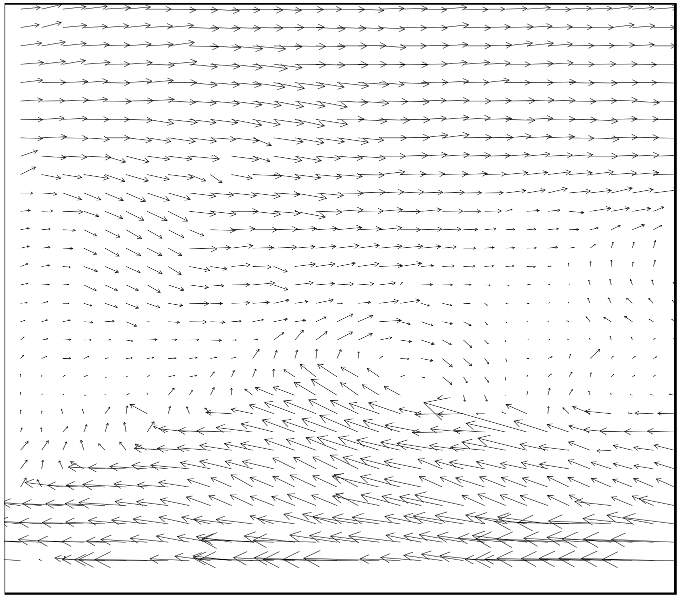


matched windows

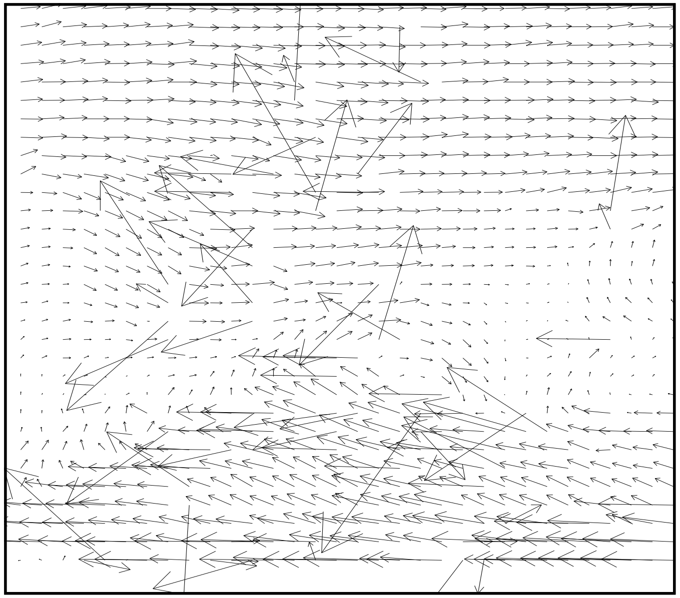


Data Validation

“article”



“lab”



Spurious or “Bad” vectors

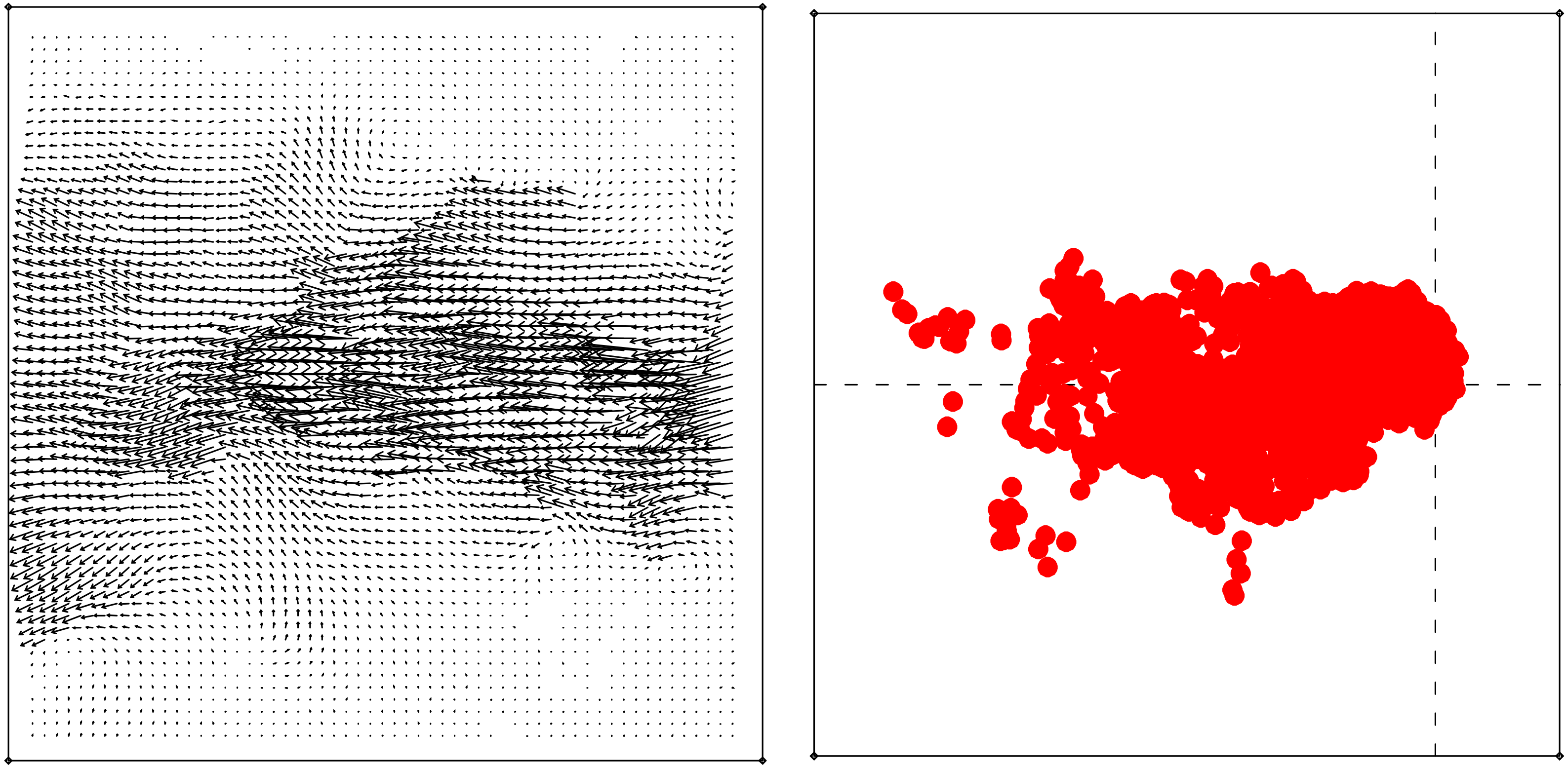
Spurious vectors

Three main causes:

- insufficient particle-image pairs
- in-plane loss-of-pairs, out-of-plane loss-of-pairs
- gradients

Effect of tracer density

N_I=80



Remedies

- increase N_I
  - practical limitations:
    - optical transparency of the fluid
    - two-phase effects
    - image saturation / speckle

- detection, removal & replacement
  - keep finite N_I ($\Gamma \sim 0.05$)
    - data loss is small
    - signal loss occurs in isolated points
    - data recovery by interpolation

Detection methods

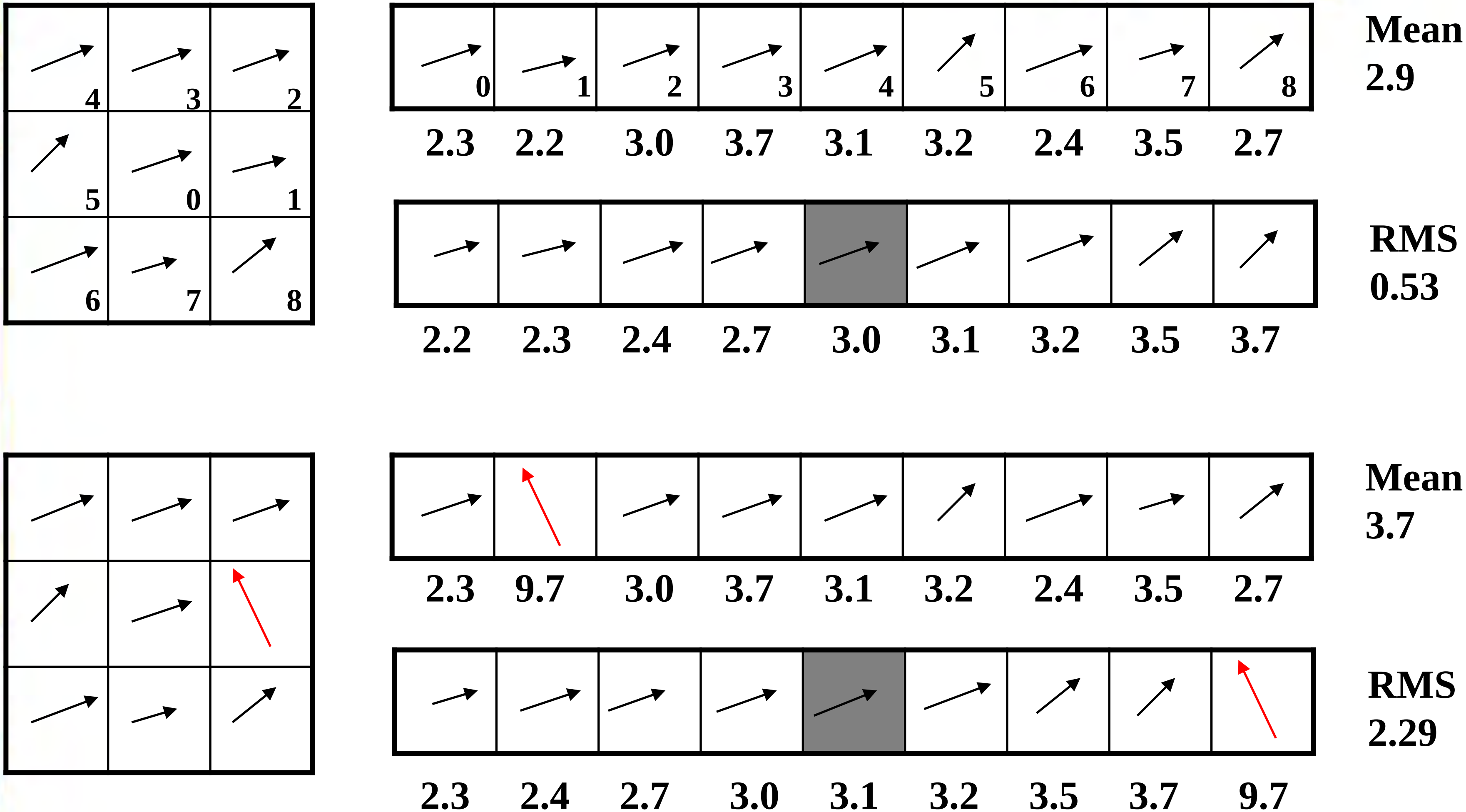
- *human perception*
- peak height
  - amount of correlated signal
- peak detectability
  - peak height relative to noise
  - lower limit for SNR
- residual vector analysis
  - fluctuation of displacement
- multiplication of correlation planes
- *fluid mechanics*
  - continuity
- *fuzzy logic & neural nets*

Residual analysis

- evaluate fluctuation of measured velocity ⇒ residual
- ideally: U_{ref} = true velocity
- reference values:
  - U_{ref} = global mean velocity
    - comparable to 2D-histogram analysis
    - does not take local coherent motion into account
    - probably only works in homogeneous turbulence
  - U_{ref} = local (3×3) mean velocity
    - takes local coherent motion into account
    - very sensitive to outliers in the local neighborhood
  - U_{ref} = local (3×3) median velocity
    - almost identical statistical properties as local mean
    - Strongly suppressed sensitivity to outliers in heighborhood

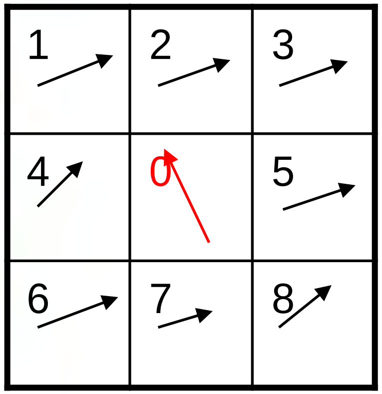
$$r = \left| U - U_{ref} \right|$$

Example of residual test



Standard mean and r.m.s. are very sensitive to bad data contamination… need robust measure of fluctuation

Median test



1 - Calculate reference velocity: median of 8 neighbors

$$u_{ref} = median\left(u_1, u_2, \ldots u_8\right)$$

2 – calculate residuals:

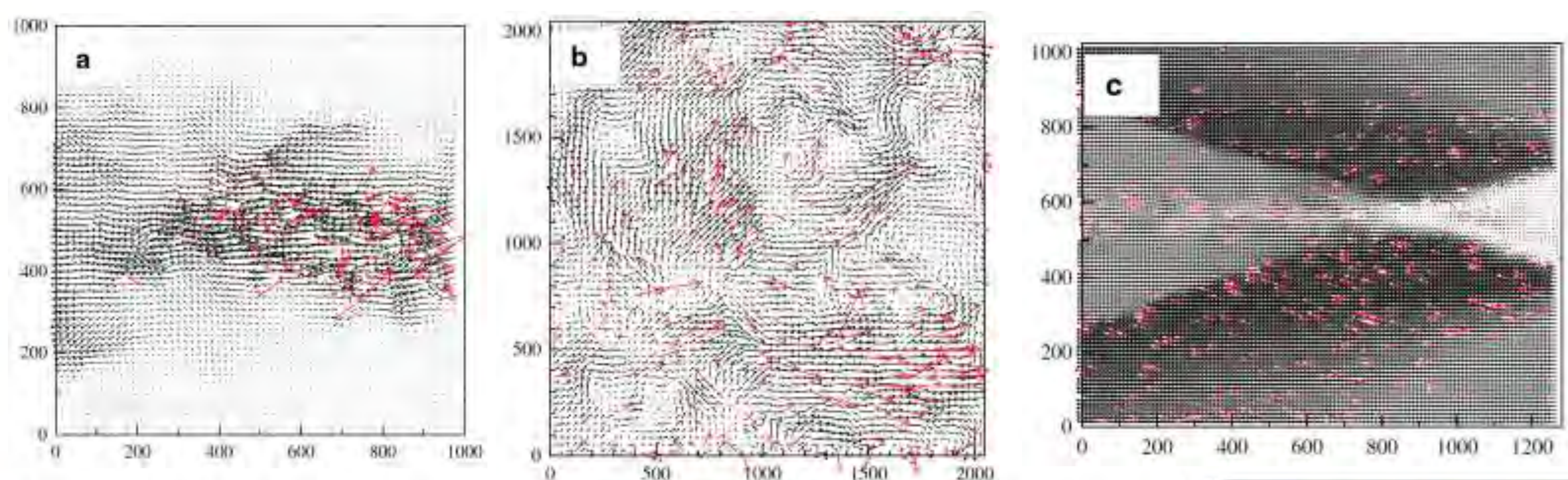
$$r_i = u_i - u_{ref}$$

3 – Normalize target residual by: median(r_i) + ε

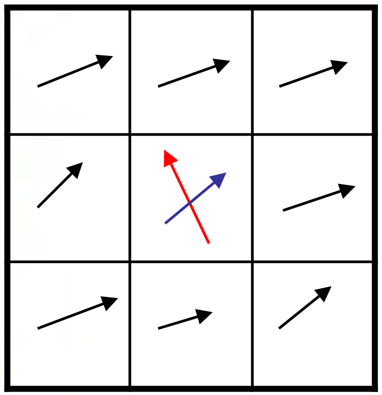
$$r_0^* = \frac{\left|u_0 - u_{ref}\right|}{median(r_i) + \varepsilon}$$

4 – Robust measure found for: $\varepsilon = 0.1$ and $r_0^* > 2$

Westerweel & Scarano, 2005



Interpolation



Bilinear interpolation satisfies continuity

For 5% bad vectors, 80% of the vectors are isolated

↓

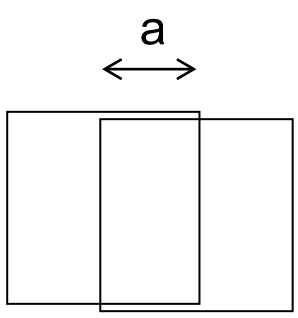
Bad vector can be recovered without any problems

N.B.: interpolation biases statistics (power spectra, correlation function)
Better not to replace bad vectors (use e.g. slotting method)

Overlapping windows

Method to increase data yield:

Allow overlap between adjacent interrogation areas



Motivation: particle pairs near edges contribute less to correlation result;
Shift window so they are in the center:
additional, relatively uncorrelated result

50% is very common, but beware of oversampling

A Generic PIV program

Data acquisition — Laser control, Camera settings, etc.

↓

Image pre-processing — Reduce non-uniformity of illumination; Reflections

↓

PIV cross-correlation — Pre-shift; Decreasing window sizes

↓

Vector validation — Median test, Search window

↓

Post-processing — Vorticity, interpolation of missing vectors, etc.

PIV software

Free

PIVware: command line, linux (Westerweel)
JPIV: Java version of PIVware (Vennemann)
MatPiv: Matlab PIV toolbox (Cambridge, Sveen)
URAPIV: Matlab PIV toolbox (Gurka and Liberzon)
DigiFlow (Cambridge), PIV Sleuth (UIUC), MPIV, GPIV, CIV, OSIV,…

Commercial

PIVtec	PIVview
TSI	Insight
Dantec	Flowmap
LaVision	DaVis
Oxford Lasers/ILA	VidPIV
…	

Particle Motion: tracer particle

- **Equation of motion for spherical particle:**

$$m_p \frac{d\vec{v}_p}{dt} = 3\pi\mu D\left(\vec{u} - \vec{v}_p\right) + \frac{1}{2} m_f \left[\frac{d\vec{u}}{dt} - \frac{d\vec{v}_p}{dt}\right] + m_f \frac{d\vec{u}}{dt} - m_p\left(1 - \frac{\rho_g}{\rho_p}\right)\vec{g}$$

Inertia | Viscous drag | Added mass | Pressure gradient | buoyancy

– Where

$m_p = \rho_p \frac{\pi D^3}{6}$, the particle mass

$m_f = \rho_f \frac{\pi D^3}{6}$, fluid mass of same volume as particle

$D =$ particle diameter

$\mu =$ fluid viscosity

$\vec{u} =$ fluid velocity

$\vec{v}_p =$ particle velocity

$\rho_g =$ fluid density

$\rho_p =$ particle material density

– Neglect: non-linear drag (only really needed for high-speed flows), Basset history term (higher order effect)

Simple thought experiment

- **Lets see how a particle responds to a step change in velocity**
  - Only consider viscous drag

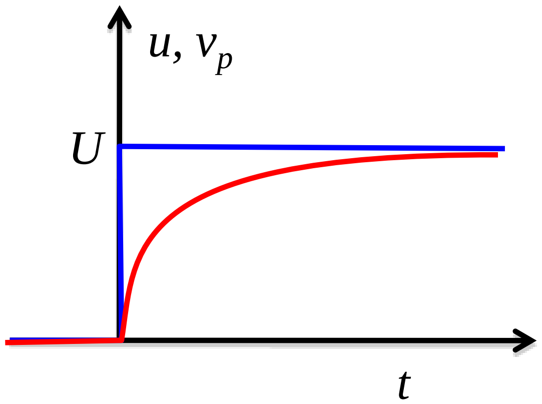
$$m_p \frac{d\vec{v}_p}{dt} = 3\pi\mu D(\vec{u} - \vec{v}_p) \qquad u = \begin{cases} 0 \text{ for } t < 0 \\ U \text{ for } t \geq 0 \end{cases}$$

$$\frac{dv_p}{dt} = \frac{18\mu}{\rho_p D^2}(U - v_p) = \frac{1}{\tau_p}(U - v_p)$$

$$v_p' + \frac{1}{\tau_p} v_p = \frac{1}{\tau_p} U$$

$$v_p = (v_p)_{\text{particular}} + (v_p)_{\text{homogeneous}} = U + C_1 \exp(-t/\tau_p)$$

$$v_p = U[1 - \exp(-t/\tau_p)]$$



Particle Transfer Function

- **Useful to examine steady-state particle response to 1-D oscillating flow of arbitrary sum of frequencies**
  - Represent *u* as an infinite sum of harmonic functions
  - Neglect gravity (DC response, not transient)

$$u(t) = \int_0^\infty \Lambda_f(\omega)\exp(i\omega t)d\omega \qquad \frac{du(t)}{dt} = \int_0^\infty i\omega\Lambda_f(\omega)\exp(i\omega t)d\omega$$

$$v_p(t) = \int_0^\infty \Lambda_p(\omega)\exp(i\omega t)d\omega \qquad \frac{dv_p(t)}{dt} = \int_0^\infty i\omega\Lambda_p(\omega)\exp(i\omega t)d\omega$$

$$\frac{2m_p}{3\pi\mu D}\frac{dv_p}{dt} = 2(u - v_p) + \frac{m_f}{m_p}\frac{m_p}{3\pi\mu D}\left[\frac{du}{dt} - \frac{dv_p}{dt}\right] + 2\frac{m_f}{m_p}\frac{m_p}{3\pi\mu D}\frac{du}{dt}$$

$$\int_0^\infty 2\frac{\rho_p D^2}{18\mu}\Lambda_p i\omega\exp(i\omega t)d\omega = \int_0^\infty 2(\Lambda_f - \Lambda_p)\exp(i\omega t)d\omega + \int_0^\infty \frac{\rho_f}{\rho_p}\frac{\rho_p D^2}{18\mu} i\omega\left[3\Lambda_f - \Lambda_p\right]\exp(i\omega t)d\omega$$

$$2iSt\Lambda_p = 2(\Lambda_f - \Lambda_p) + i\gamma St\left[3\Lambda_f - \Lambda_p\right] \qquad St = \frac{\tau_p}{\tau_f} = \frac{\rho_p D^2\omega}{18\mu} \qquad \gamma = \frac{\rho_f}{\rho_p}$$

Particle Transfer Function

- **This can be rearranged:**

$$\left\|\frac{\vec{v}_p}{\vec{u}}\right\| = \left[\frac{\Lambda_p \Lambda_p^*}{\Lambda_f \Lambda_f^*}\right]^{1/2} = \left[\frac{A^2 + B^2}{A^2 + 1}\right]^{1/2}$$

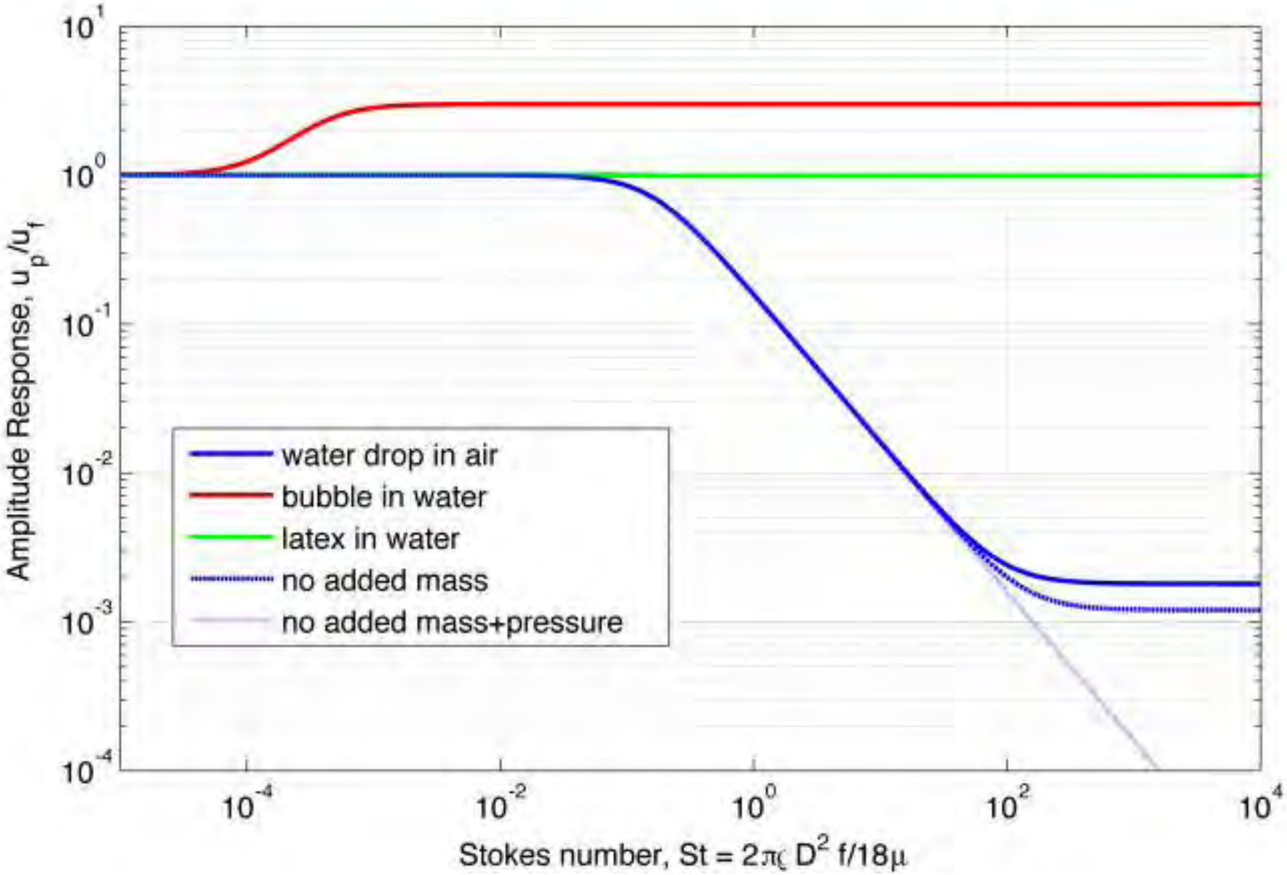
$$A = \frac{2}{St\,(1+\gamma)}$$

$$\phi = \tan^{-1}\left[\frac{\mathrm{Im}(\Lambda_p / \Lambda_f)}{\mathrm{Re}(\Lambda_p / \Lambda_f)}\right] = \tan^{-1}\left[\frac{A(1-B)}{A^2 + B}\right]$$

$$B = \frac{3\gamma}{2+\lambda}$$

- **Examine**
  - Liquid in air, gas in water, plastic in water

Liquid particles in Air



- Liquid drops in air require St~0.3 for 95% fidelity (1 μm ~ 15 kHz)
- Size relatively unimportant for near-neutrally buoyant particles
- Bubbles are a poor choice: always overrespond unless quite small